Staleness Bounds and Efficient Protocols for Dissemination of Global Channel State Information

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***Abstract*— This paper considers the problem of achieving global channel knowledge throughout a fully connected packetized wireless network with time-varying channels. While the value of channel state information at the transmitter (CSIT) is now well known, there are many scenarios in which it is helpful to have additional channel knowledge beyond conventional CSIT, e.g., cooperative communication systems. The overhead required for global CSI knowledge can be significant, particularly in time-varying channels where the quality of channel estimates is dominated by the "staleness" of the CSI. Nevertheless, the fundamental limits and feasibility of tracking global CSI throughout a network have not been sufficiently studied. This paper presents a framework for analyzing the staleness of protocols that estimate and disseminate CSI to all nodes in a fully connected network. Fundamental bounds on achievable staleness are derived, and efficient dissemination protocols are developed, which achieve these limits. The results provide engineering guidelines on the feasibility of tracking global CSI as a function of network size, the size and composition of the packets, packet error rate, and channel coherence time.**

***Index Terms*— Age of information, global channel state information (CSI), optimal scheduling, data dissemination protocols.**

I. INTRODUCTION

IN wireless networks, knowledge of channel state information (CSI) by the nodes in the network can often be used to improve one or more performance characteristics of the network, e.g., increase data rates, reduce interference, and/or improve energy efficiency. In point-to-point links, knowledge of the channel state information at the transmitter (CSIT) can improve performance through techniques such as Tomlinson-Harashima precoding [1], waterfilling [2], [3], and/or adaptive transmission over fading channels [4]. In multiple-input multiple-output (MIMO) channels, CSIT allows for coherent transmission techniques like beamforming and can also provide multiplexing gains [5]–[7]. CSIT can also be used in MIMO systems for interference mitigation, e.g., zero-forcing beamforming [7], nullforming [8], and interference alignment [9].

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While the value of CSIT is well-established in the literature, there are also many examples of systems where the nodes in the wireless network benefit from having a more comprehensive view of the channel states in the network beyond just CSIT. For example:

1) Cooperative relaying. Optimum power allocation and/or relay selection strategies generally require the source to know the magnitude of the relay-destination channels [10]–[12]. In multi-relay systems, optimum transmission schemes may require the relays to know all source-relay and relay-destination channels [13]. In cooperative networks with dynamic relay pairing, stable matchings require global channel state knowledge [14].
2) Distributed communication systems. While distributed beamforming can be achieved with CSIT [15], [16], more general distributed transmission schemes such as zero-forcing beamforming [17], nullforming [18], and interference alignment [19], [20] require each transmitting node to know all source-destination channels to be able to compute the desired precoding vector. Similarly, optimum combining in distributed reception systems requires the fusion center to know all source-destination channels [21], [22].
3) Cross-layer design for multihop networks. Optimization of the end-to-end data rate and scheduling in multihop wireless networks generally requires global CSI [23], [24]. Routing performance in multihop networks is also significantly improved with global CSI [25], [26].

Global knowledge of CSI also permits nodes to opportunistically determine an appropriate network structure and communication strategy for efficient operation under the current channel state. Furthermore, availability of global CSI allows nodes to change roles over time, perhaps participating in a coherent transmit cluster at one point in time, then serving as a relay at another point in time, with the current role dynamically determined by the evolving global channel state.

This paper considers theoretical bounds and efficient protocols for estimation and dissemination of global CSI in N-node wireless networks with reciprocal channels. By "global CSI", we mean that each node maintains its own table of estimates for *all* $L = \frac{N(N-1)}{2}$ reciprocal channels in the network, not just the $N-1$ channels to

TABLE I
ACHIEVABLE STALENESS METRICS

protocol type	estimates disseminated per packet	maximum staleness	average staleness
deterministic	1	$\frac{N^2-N+2}{2}(D+1)$	$\frac{N^3-4N^2+13N-14}{4(N-1)}(D+1)$
	$N-1$	$(N-1)(D+N-1)$	$\frac{2N^2-2N-1}{3N}(D+N-1)$
random [38]	1	-	$\frac{N(N-1)}{2}(D+1)$
	$N-1$	-	$\frac{3N-4}{2}(D+N-1)$

which a given node is directly connected. Nodes obtain estimates of channels to which they are not directly connected via CSI "dissemination". Specifically, nodes disseminate CSI by embedding one or more CSI estimate(s) in each transmission so other nodes can learn the states of channels to which they are not directly connected. Over time, each node in the network directly estimates the $N-1$ channels to which it is directly connected and "indirectly estimates" the remaining $L-N+1$ channels in the network by collecting disseminated CSI. While some recent studies have considered the problem of estimating and tracking so-called "global CSI", e.g., [27]–[29], the notion of global CSI in these papers is not the same as the notion of global CSI considered here. In that prior work, the roles of the nodes are fixed and the focus is on providing estimates of all transmit-receive channels to all transmit nodes in the network, i.e., global *CSIT*. We emphasize that our notion of global CSI does not presume roles for the nodes in the network and allows nodes to dynamically adapt their roles by estimating and tracking *all* of the L reciprocal channels in the network.

The focus of this paper is on wireless networks with reciprocal channels. In this setting, there are two sources of error in the global CSI at each node in the network: (i) channel estimation error, typically governed by fundamental bounds such as the Cramer-Rao lower bound (CRLB) and (ii) error caused by time-variation and staleness, i.e., the delay from when the time-varying channel was estimated and the current time n. In this paper, and as in the recent work on "age of information" [30]–[32], we assume the type (ii) error is dominant. The value of stale (sometimes called delayed or outdated) CSIT has been considered only recently in [33]–[35]. While it was shown that even completely stale CSIT can still be useful in certain scenarios [33], there is generally a loss of performance with respect to perfect CSI knowledge as CSI becomes more stale [35]. Moreover, the focus of these staleness studies has been on maximizing degrees of freedom in conventional MIMO channels with delayed CSIT, and not on cooperative, distributed, or multihop scenarios where a more comprehensive knowledge of CSI is necessary. In these types of scenarios, performance can be highly sensitive to the accuracy of the CSI, which is directly related to its staleness [12], [30], [36], [37].

The main contributions of this paper are as follows. First, we develop a new and general framework for quantifying the staleness of global CSI in packetized wireless networks. This framework accounts for the number of channel states disseminated in each packet as well as the additional data and overhead in each packet which contribute to the staleness of the CSI. We then develop bounds on the minimum achievable CSI staleness throughout a network (Theorems 1, 2, 4, and 5). Subsequently, we develop CSI dissemination protocols and quantify the maximum and average staleness of these protocols as a function of the number of nodes in the network and composition of each packet. We show these protocols are *efficient* in the sense that they achieve maximum and average staleness within a small, constant gap of the explicit lower bounds (Theorems 3 and 6). We compare our results with other, recently reported results on "random" or opportunistic CSI dissemination protocols [38] as well, and the results in this paper provide an explicit characterization of the staleness penalty of using opportunistic protocols with respect to the "best case" CSI dissemination protocols developed here. The results in this paper provide explicit efficient protocols for disseminating CSI in wireless networks as well as engineering guidelines on the feasibility of tracking global CSI in terms of the network size, packet size and composition, and channel coherence time. While the case of partial network connectivity is an important future extension of this work, the fully-connected case considered here is a canonical network topology for developing the staleness framework, and serves as a reference with which future work can be compared. Since the total number of channels in fully-connected network is $L=\frac{N(N-1)}{2}$ and each one of the N nodes maintains its own table of L CSI estimates, the total number of CSI estimates throughout a fully-connected network is equal to $LN=\frac{N^3-N^2}{2}$. Since this quantity scales as $O(N^3)$, the cost of estimating global CSI can be prohibitive for large N. Nevertheless, there is a gap in solidly understanding the overhead and tradeoffs involved in tracking global CSI throughout a network, particularly in cases where N is small and global CSI may be feasible.

Table I summarizes the achievable staleness for the deterministic protocols presented here, as well as the random/opportunistic protocols presented in [38], and includes both the case where a single CSI estimate is disseminated in each packet, as well as the case where a node's entire table of $N-1$ direct channel estimates is disseminated in each packet. The staleness units are in number of words, where it is assumed each packet consists of D words of overhead and data, and a single disseminated channel estimate requires one word.

The rest of this paper is organized as follows. Section II describes the system model and provides definitions and examples of the maximum and average staleness metrics considered in this paper. Section III derives lower bounds on the maximum and average staleness, develops efficient deterministic CSI dissemination protocols that achieve these bounds within a small, constant gap, and presents several extensions to the model as well as suggestions for potential future studies. Section IV provides numerical results demonstrating the staleness of the CSI dissemination protocols, while Section V provides conclusions. Proofs of the major theorems are provided in the Appendices.

Notation: We adopt the notation used in the graph theory literature, letting K_N denote the complete graph with N vertices, and when N is even we let $K_N - I$ denote the complete graph with a 1-factor removed. We use standard cycle notation to describe permutations [39], where each number in parentheses is sent to the one immediately following it, and the last number in parentheses is sent back to the first. For the permutation $\sigma = (1, 2, 3)(4)$, for example, $\sigma(1) = 2$, $\sigma(2) = 3$, $\sigma(3) = 1$, and $\sigma(4) = 4$. In addition, $\sigma^n(\cdot)$ permutes the input n times, so for example $\sigma^2(1) = 3$. For a sequence i_n indexed by integer n, the shorthand $i_{n_1:n_2}$ denotes the sequence $i_{n_1}, i_{n_1+1}, \ldots, i_{n_2}$ where $n_1 \leq n_2$. If j_n is also a sequence indexed by n, then $(i, j)_n$ is shorthand for the pair (i_n, j_n).

II. System Model

We consider a system model consisting of a cluster of N connected wireless nodes communicating over a time-varying channel. For simplicity we focus on the case where each node has a single antenna, and we denote the complex channel gain between nodes i and j at time n as $h_{i,j}[n]$. We assume time-division duplexing (TDD) and therefore the channels between all nodes are reciprocal so $h_{i,j}[n] = h_{j,i}[n]$; with this assumption, the network consists of $L \triangleq (N^2 - N)/2$ complex channel gains between all pairs of nodes. We assume that, due to the choice of underlying communication scheme (e.g., interference alignment, distributed MIMO, etc.), each of the N nodes in the network requires global channel state information, and therefore each requires knowledge of all L complex channel gains. Estimating and disseminating these channel gains throughout the network is the problem of interest here.

We assume fixed-length packet transmissions between the nodes in the network of the form shown in Fig. 1. Each packet is assumed to contain overhead, data, and disseminated CSI corresponding to M channel estimates that the transmitting node disseminates to the other nodes in the network. In general, the CSI dissemination portion of the packet provides a means for node k to receive an estimate of the (i, j) channel when $i \neq j \neq k$ since node k has no means for directly estimating channels to which it is not connected. Each disseminated channel estimate is assumed to have a length of one word, where we define a *word* as a general time unit representing the amount of time required to transmit a single CSI estimate. By adopting words as the basic time unit, the staleness framework developed in this paper can be applied to a wide range of wireless communication settings and standards, with widely different choices of system parameters, packet structures, and data rates. The data and overhead are assumed to have a length of D words, hence the total packet length is $P = M + D$ words. While Fig. 1 shows a particular packet structure, the position of the overhead, data, and disseminated CSI within any packet is not consequential in our analysis.

Fig. 1. Example fixed-length packet showing overhead, data, and CSI dissemination. The CSI dissemination consists of $M \in \{1, \ldots, N-1\}$ channel estimates where the length of each channel estimate is one word. The data and overhead consist of D words. The total packet length is $P = M + D$ words.

Under the fully-connected network assumption, when node i transmits a packet at time n, all other nodes $j \neq i$ receive the packet[1] similar to the model in [41], and so only one node can transmit at a time. Each node $j \neq i$ is assumed to receive the packet reliably, and each node subsequently does two things:

1) Estimates the channel $h_{i,j}[n]$, which can be obtained via a known training sequence in the packet, e.g., a known preamble embedded in the overhead, and/or through blind channel estimation techniques.
2) Extracts disseminated CSI and uses it to replace any CSI in its local table that is "staler".

Each node maintains its own table of estimates of the current state of all L channels in the network. We denote the k^{th} node's estimate of the (i, j) channel during the packet transmitted at time n as $\hat{h}_{i,j}^{(k)}[n]$. Note that $N - 1$ of a node's estimates are *directly* obtained via channel estimation in step 1 above (for $i = k$ or $j = k$). The remaining $L - N + 1$ estimates are *indirectly* obtained via disseminated CSI in step 2 above (for $i, j \neq k$). Thus, the network contains a total of $N(N-1) = 2L$ directly estimated parameters, and $N(L - N + 1) = L(N - 2)$ indirectly estimated parameters.

To illustrate the basic concepts, consider an $N = 3$ node network and assume each node disseminates a single CSI estimate per packet ($M = 1$) with no overhead or data ($D = 0$). We refer the reader to Table II as well as Fig. 2 for a graphical representation of this example. The following is a description of a deterministic packet transmission sequence and the resulting CSI estimates at each node in the network:

$n = 0$:

Suppose the first packet is transmitted by node 1. Prior to the first packet transmission, there is no knowledge of CSI anywhere in the network. As such, the first packet ($n = 0$) can only be used for estimation, and not dissemination. In this case, node 2 directly estimates the channel $h_{1,2}[0]$ (computing the estimate $\hat{h}_{1,2}^{(2)}[0]$) and node 3 directly estimates the channel $h_{1,3}[0]$ (computing the estimate $\hat{h}_{1,3}^{(3)}[0]$). The

[1]The packet from node i may have encrypted data directed to a particular node but we assume all nodes can receive the packet due to the broadcast nature of the wireless network. To mitigate possible erroneous dissemination of CSI by adversarial nodes, the transmitters can cryptographically sign the CSI with a private key such that each receiver can verify the CSI is transmitted by a trusted node using a known public key [40].

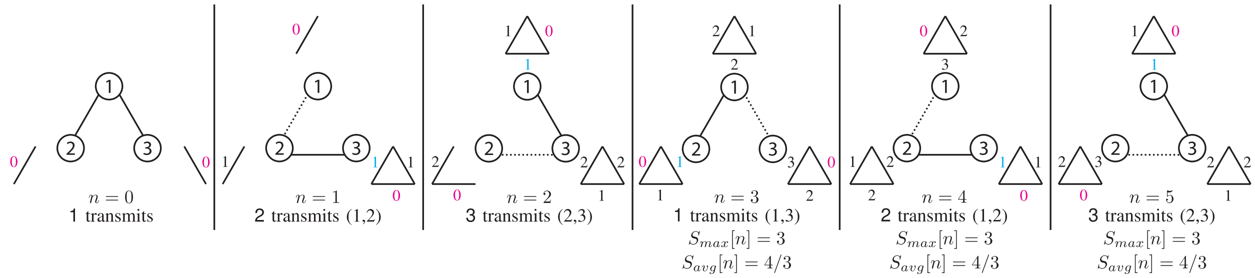

Fig. 2. Operation of protocol for 3 node case. Numbers on edges indicate staleness of each channel estimate locally at each node; red numbers indicate CSI estimates have been refreshed through direct estimation, blue numbers indicate CSI refreshed through dissemination, and black numbers indicate no update to CSI since the last packet.

staleness of both of these estimates is zero since they were obtained directly from the current packet.

$n = 1$:

Suppose this packet is transmitted by node 2. Since node 2 now has an estimate of the $(1, 2)$ channel, it disseminates $\hat{h}^{(2)}_{1,2}[0]$ in its packet. Node 1 directly estimates the channel $h_{1,2}[1]$ (computing the estimate $\hat{h}^{(1)}_{1,2}[1]$) and node 3 directly estimates the channel $h_{2,3}[1]$ (computing the estimate $\hat{h}^{(3)}_{2,3}[1]$). Additionally, node 3 extracts the disseminated CSI $\hat{h}^{(2)}_{1,2}[0]$ since it does not have a prior estimate of the $(1, 2)$ channel. Node 1 does not use the disseminated CSI since it is staler than the direct estimate $\hat{h}^{(1)}_{1,2}[1]$. To summarize, and as shown in Fig. 2, after node 2 transmits its packet, node 1 has a current estimate of the $(1, 2)$ channel, node 2 has a stale estimate of the $(1, 2)$ channel, and node 3 has stale estimates of the $(1, 2)$ and $(1, 3)$ channels as well as a current estimate of the $(2, 3)$ channel. Estimates of all channels in the network now exist at node 3, but two more packets are required for node 1 and node 2 to each have complete CSI tables.

$n = 2$:

Assume this packet is transmitted by node 3 and that node 3 disseminates $\hat{h}^{(3)}_{2,3}[1]$. Node 1 directly estimates the channel $h_{1,3}[2]$ (computing the estimate $\hat{h}^{(1)}_{1,3}[2]$) and node 2 directly estimates the channel $h_{2,3}[2]$ (computing the estimate $\hat{h}^{(2)}_{2,3}[2]$). Additionally, node 1 extracts the disseminated CSI $\hat{h}^{(3)}_{2,3}[1]$. Node 1 now has a complete CSI table.

$n = 3, \ldots$:

Assume the nodes repeat this same sequence of steps which are summarized in the 3-round protocol shown in Table II. This periodic 3-round protocol is shown in Fig. 2, where the time of the most recent information is indicated locally on each of the figures for each node.

While the previous example considered the $M = 1$ case where each node disseminates a single channel state, this framework also allows for multiple CSI estimates to be disseminated per packet. It is important to note, however, that nodes should only disseminate directly estimated channels since any indirectly estimated channel, i.e., a channel learned via dissemination, is at least as stale as the CSI at all other nodes in the fully-connected network. Hence, we restrict attention to protocols with $M \in \{1, \ldots, N-1\}$.

TABLE II
3-NODE DETERMINISTIC PROTOCOL

time	transmitting node	disseminated channel
$n = 0, 3, 6, \ldots$	1	(1,3)
$n = 1, 4, 7, \ldots$	2	(1,2)
$n = 2, 5, 8, \ldots$	3	(2,3)

The following definitions formalize the staleness metrics considered in the remainder of this paper.

Definition 1 (Staleness): The staleness $s^{(k)}_{i,j}[n]$ of the CSI estimate $\hat{h}^{(k)}_{i,j}[n']$ at time $n \geq n'$ is $(n - n')P$ words.

In the three-node example at time $n = 5$, node 2 has the CSI estimate $\hat{h}^{(2)}_{1,2}[3]$. Hence, the staleness of node 2's estimate of the $(1, 2)$ channel at time $n = 5$ is $s^{(2)}_{1,2}[5] = (5 - 3)P = 2$ since the packet length $P = 1$ in this example. In general, the staleness obeys

$$s^{(k)}_{i,j}[n] = \begin{cases} 0 & \text{direct estimate is made} \\ s^{(k')}_{i,j}[n] & (i, j) \text{ estimate disseminated} \\ & \text{ by node } k', \text{ where } k \neq k' \\ s^{(k)}_{i,j}[n-1] + P & \text{otherwise.} \end{cases}$$

Recall that a node cannot make observations while transmitting. Hence, direct estimation of the (i, j) channel occurs at node k for $k = i$ when node j transmits (indicated by red numbers in Fig. 2), indirect estimation of the (i, j) channel occurs at node k when the (i, j) channel is disseminated by node $k' \neq k$ for $i, j \neq k$ (indicated by blue numbers in Fig. 2), and CSI estimates simply grow staler by one packet when no estimates of the (i, j) channel are made (indicated by black numbers in Fig. 2).

We define a *protocol* as a sequence of transmitting nodes and the channel indexes they disseminate, as shown by the example periodic 3-round protocol in Table II. In general, protocols can be deterministic or random, periodic or aperiodic. For deterministic protocols considered in this paper, we define a maximum staleness metric.

Definition 2 (Maximum Staleness): The maximum staleness S_{max} of a deterministic protocol is defined as

$$S_{max} = \max_{i,j,k,n \geq \bar{n}} s^{(k)}_{i,j}[n] \tag{1}$$

for $\bar{n}$ sufficiently large such that all nodes have complete CSI tables.

In the three node example, note that all nodes have complete CSI tables at $\bar{n} = 3$. Observe that the stalest individual CSI value is always 3 for all $n \geq \bar{n}$. Hence, the maximum staleness is $S_{max} = 3$ for the three-node example.

The tolerable maximum staleness can be related to the channel coherence time. For example, if a particular communication system requires T seconds to transmit one word (and PT seconds to transmit one packet), then TS_{max} would need to be less than the channel coherence time in order for all disseminated CSI in the network to be accurate at all times.

Finally, we define an average staleness metric. This metric applies not only to deterministic periodic protocols but also random protocols satisfying certain mild conditions [38].

Definition 3 (Average Staleness): The average staleness S_{avg} of a protocol is defined as

$$S_{avg} = \frac{1}{LN} \mathrm{E}\left[\sum_{i,j,k} s_{i,j}^{(k)}[n]\right] \tag{2}$$

where the expectation is over $n \geq \bar{n}$ for $\bar{n}$ sufficiently large such that all nodes have complete CSI tables.

At times, it is convenient to consider the instantaneous maximum and average staleness, denoted $S_{max}[n]$ and $S_{avg}[n]$, respectively, which have the same definitions as (1) and (2), but without taking the maximum or average over time n. In addition, we use the subscript *direct* or *indirect* to distinguish between the staleness of those $2L$ parameters which are *directly* estimated (i.e., $s_{i,j}^{(k)}[n]$ for $i = k$ or $j = k$), and those $L(N-2)$ parameters which are *indirectly* estimated (i.e., $s_{i,j}^{(k)}[n]$ for $i, j \neq k$).

In the three node example, for $\bar{n} = 3$, observe that the average staleness is $S_{avg} = \frac{4}{3}$. In fact, the instantaneous average staleness $S_{avg}[n]$ is constant for all $n \geq \bar{n}$. We claim the deterministic periodic protocol in Table II is an efficient deterministic protocol for the 3-node case in terms of minimizing maximum staleness *and* minimizing average staleness. The next section formalizes this claim and generalizes the design of efficient deterministic protocols to K_N.

III. Deterministic CSI Dissemination Protocols

In this section we develop theoretical staleness bounds and explicit deterministic protocols for efficient dissemination of global CSI. The deterministic protocols developed in this section specify a specific order of node transmission and CSI dissemination. Hence, in contrast to the random protocols considered in [38], these protocols are effectively driven by the CSI dissemination process.

We say that a deterministic protocol is a *valid* protocol if the deterministic sequence of CSI dissemination results in a state where every node eventually has a complete CSI table. Clearly, there are an infinite number of such protocols, but we observe there are two conditions which must be met for a protocol to be valid:

1) Each node must transmit at least once.
2) Each of the L channel gains in the network must be disseminated at least once.

If either of these two conditions are not met, the network cannot have a complete CSI table and the protocol cannot be valid. For example, if node i never transmitted, then any other node $j \neq i$ would not be able to estimate (or receive through dissemination) the (i, j) channel. Similarly, if the (i, j) channel was never disseminated, then node $k \neq i \neq j$ would not have an estimate of the (i, j) channel.

Recall that $M \in \{1, \ldots, N-1\}$ denotes the number of CSI parameters disseminated in a given packet. We analyze CSI dissemination protocols in the two extremes: (i) the $M = 1$ case where a single CSI parameter is disseminated in each packet, and (ii) the $M = N-1$ case where all directly estimated CSI parameters are disseminated in each packet.

A. Transmitting Node Disseminates a Single CSI Estimate

The following two theorems establish lower bounds on the maximum and average staleness of valid protocols for the case when a single CSI estimate is disseminated in each packet.

Theorem 1 (Lower Bound on Maximum Staleness for $M = 1$): For $M = 1$, the maximum staleness S_{max} of any valid protocol is lower bounded by $S_{max} \geq S_{max}^* = L(D+1) = \frac{N(N-1)}{2}(D+1)$. Moreover, any protocol which achieves this bound must be L-periodic.

Proof: Recall that disseminated estimates are always at least 1 packet old (i.e., have staleness $\geq P$). Every one of the L channel gains must be indirectly estimated by some nodes. Since it requires L packets to disseminate all L channel gains, and they each have staleness at least P at the time of dissemination, $S_{max} \geq S_{max}^* = LP = L(D+1)$.

To see that any protocol achieving this bound must be L-periodic, assume that at time $n = n_0$ we have $S_{max}[n_0] = S_{max}^*$. This implies that the last L packets must have each disseminated the L distinct channel gains, each estimate having staleness P (i.e., the freshest possible staleness of a disseminated estimate). To maintain maximum staleness $S_{max}[n_0 + 1] = S_{max}^*$ at time $n_0 + 1$, the channel estimate which was disseminated at time $n_0 - L$ (i.e., the stalest CSI) must be disseminated next; otherwise, we would have $S_{max}[n] = S_{max}^* + P$. Continuing this argument at time $n_0 + 2$, we see that the protocol repeats and has period L. ■

Theorem 2 (Lower Bound on Average Staleness for $M = 1$): For $M = 1$, the average staleness S_{avg} of any valid protocol is lower bounded by $S_{avg} \geq S_{avg}^* = \frac{N^3 - 3N^2 + 8N - 8}{4N}(D+1)$.

Proof: Recall that disseminated estimates are always at least 1 packet old (i.e., have staleness $\geq P$). When node i disseminates the (i, j) channel to all nodes, the $N-1$ other nodes refresh their *direct* estimates of the channel between themselves and node i, yielding a staleness of zero for those $N-1$ direct channel estimates. These $N-1$ other nodes also refresh their indirect estimates of the disseminated (i, j) channel to have a staleness of at least P. Node j prefers the fresher, direct estimate during i's transmission since it has staleness zero; thus, after any packet transmission, only $N-2$ of the estimates are refreshed due to dissemination. In summary, any transmission in the network results in $N-1$ of the LN channel estimates being refreshed to have staleness zero, and $N-2$ of the LN channel estimates being refreshed to have staleness at least P.

Consider just the $L(N-2)$ indirect channel estimates which must be updated through dissemination. Recall that with $M = 1$, only a single CSI estimate can be disseminated in each packet. Thus, at any given state, there are $N-2$ of them with at least staleness P, $N-2$ of them with staleness $2P$, ..., and $N-2$ of them with staleness LP and so $S_{avg,indirect} \geq (L+1)P/2$.

From Lemma 2, the average staleness of the $2L$ direct channel estimates which can be directly estimated satisfies $S_{avg,direct} \geq (N-1)P/2$. Hence, the weighted average gives us a bound on the average staleness as

$$S_{avg} \geq \frac{L(N-2)S_{avg,indirect} + 2LS_{avg,direct}}{LN} = \frac{(N^3 - 3N^2 + 8N - 8)}{4N}P.$$

■

To facilitate the development of deterministic CSI protocols that achieve or approach these lower bounds within a constant gap, we adopt the following definition of an efficient protocol.

Definition 4 (Efficient Protocol): An efficient deterministic protocol is a valid protocol that simultaneously achieves maximum staleness $S^*_{max} + k_{max}P$, while also achieving an average staleness of at most $S^*_{avg} + k_{avg}P$ for constants k_{max}, k_{avg} and all N.

While ideally we would employ protocols with $k_{max} = k_{avg} = 0$ that achieve the staleness lower bounds with equality, such protocols do not exist for all choices of N as we will see. In such cases, the best we can do is to be within a constant that is independent of N.

Since nodes only disseminate directly estimated channels, we can represent any valid protocol for $M = 1$ as traveling along edges of the complete graph K_N. That is, traveling along the edge from vertex i to j represents dissemination of the (i, j) channel estimate by node j. The conditions above for a protocol to be valid can be rewritten, respectively, in terms of a sequence of edge traversals of K_N:

1) Each vertex must be traveled to at least once.
2) Each edge of the graph must be traversed at least once.

Since the problem of finding efficient deterministic protocols relies heavily on traversals of K_N, it is convenient to divide the efficient deterministic protocol design construction into two cases: (1) N is odd, (2) N is even.

1) When N Is Odd: To achieve the bound of Theorem 1, an approach suggested by the proof is to always disseminate the freshest CSI. This is equivalent to stating the edge activations in K_N must be adjacent, so the sequence is a *walk*. Over any sequence of L disseminations, an efficient protocol would not disseminate the same channel estimate twice, which suggests we seek a walk on K_N that visits every edge exactly once. Such a walk is called an Eulerian tour [42], and such tours provide a method for constructing protocols that achieve the bound of Theorem 1, as we will show in Theorem 3 below. Since Eulerian tours can be constructed quite readily using, for example, Fleury's algorithm [43], we have an explicit construction for valid protocols that achieve the maximum staleness lower bound of Theorem 1.

Turning attention to the average staleness lower bound in Theorem 2, we note that an arbitrary Eulerian tour found by applying Fleury's algorithm does not necessarily result in a protocol with average staleness close to the bound. In each L-period (corresponding to one Eulerian tour), each of the N nodes transmits $(N-1)/2$ times. If, for example, a node's $(N-1)/2$ transmissions are all clustered at the beginning of the L-period, the node will subsequently not transmit for a long period of time. During that node's period of silence, all other nodes' *direct* estimates to that silent node become quite stale, thereby resulting in poor average staleness. Intuitively, average staleness is improved by finding Eulerian tours that uniformly distribute every node's transmissions throughout the L-periodic transmission sequence. The following explicit L-periodic protocol realizes this idea through the use of the Lucas-Walecki construction [44], [45] for odd N, which guarantees the L-periodic Eulerian tour on K_N has the additional structure that every node transmits exactly once in each block of N packets.

Protocol 1 ($M = 1$ and N odd).

Node transmission order follows an Eulerian tour composed of $(N-1)/2$ edge-disjoint Hamiltonian cycles of K_N. Let H_0 be the length N zig-zag Hamiltonian cycle

$$H_0 = \left\{1, N-1, 2, N-2, .., \frac{N-1}{2}, \frac{N+1}{2}, N\right\}$$

and let σ be the permutation whose disjoint cycle decomposition is

$$\sigma = (1, 2, \ldots, N-1)(N).$$

Let $H_m = \sigma^m(H_0)$ which forms a Hamiltonian cycle decomposition of K_N for $m = 0, 1, \ldots, (N-3)/2$.

Since the protocol repeats with period L, consider only times $0 \leq n \leq L-1$. At time n, node i_n disseminates its estimate of the (i_n, j_n) channel. Let $j_n = i_{n-1}$ so node i_n always disseminates the freshest CSI, i.e. an estimate of the channel between itself and the last node that transmitted. The transmitting node at each time n is given by

$$i_{mN:mN+N-1} = \sigma^m(H_0).$$

For example, if node i_n disseminates its estimate of the (i_n, j_n) channel at time n, the length $L = 21$ periodic protocol for $N = 7$ is given by

$$i_{0:L-1} = 1, 6, 2, 5, 3, 4, 7 \mid 2, 1, 3, 6, 4, 5, 7 \mid 3, 2, 4, 1, 5, 6, 7$$
$$j_{0:L-1} = 7, 1, 6, 2, 5, 3, 4 \mid 7, 2, 1, 3, 6, 4, 5 \mid 7, 3, 2, 4, 1, 5, 6$$

where we note that each node transmits $(N-1)/2 = 3$ times, and each node transmits once per N-block. The efficiency of Protocol 1 is established below in Theorem 3.

2) When N Is Even: Recall that a connected graph admits an Eulerian tour if and only if every vertex has even degree [42]. When N is even, all vertices in the complete

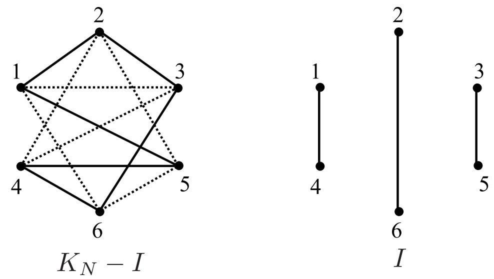

Fig. 3. Partition of K_N into $K_N - I$ and I for $N = 6$.

graph K_N have odd degree, hence an Eulerian tour does not exist and always disseminating the freshest CSI is not possible. As such, the maximum and average staleness bounds in Theorems 1 and 2 are not achievable when N is even.

Nevertheless, an efficient protocol can be constructed by permitting dissemination of CSI that was estimated two packets ago. Specifically, for one period of an L-periodic protocol, dissemination of "less-fresh" CSI is only needed for $N/2$ of the L transmissions. To formalize this notion on the graph, we observe that by subtracting a 1-factor (corresponding to any $N/2$ edges between distinct pairs of vertices) from K_N, the degree of each vertex of the resulting subgraph $K_N - I$ is even. Thus, similar to K_N for odd N, the subgraph $K_N - I$ for even N admits an Eulerian tour composed of edge-disjoint Hamiltonian cycle decompositions and $L - N/2$ channel estimates can be disseminated with staleness P. The $N/2$ missing edges from the 1-factor I, however, also need to be disseminated; inserting them back into the sequence disrupts the Eulerian tour, requiring dissemination of $N/2$ estimates with staleness $2P$. An example subgraph $K_N - I$ with missing edges of the 1-factor is shown in Fig. 3 for $N = 6$, where one of the two edge-distinct Hamiltonian cycles of $K_N - I$ is indicated by the solid lines, and the other by dashed lines.

Based on these ideas, we now give a formal description of an efficient protocol for the case where N is even.

Protocol 2 ($M = 1$ and N even).
Step 1: Construct Eulerian tour of $K_N - I$ composed of edge-disjoint Hamiltonian cycle decompositions.

Let H_0 be the length N zig-zag Hamiltonian cycle

$$H_0 = \left\{1, 2, 3, N, 4, N-1, .., \frac{N}{2}+3, \frac{N}{2}+1, \frac{N}{2}+2\right\}$$

and let σ be the permutation whose disjoint cycle decomposition is

$$\sigma = (1)(2, \ldots, N).$$

Let $H_m = \sigma^m(H_0)$ which forms a Hamiltonian cycle decomposition of $K_N - I$ for $m = 0, 1, \ldots, N/2 - 2$.

Construct an intermediate periodic protocol that disseminates channel estimates corresponding to edges of $K_N - I$, and therefore has period $L - N/2$. The sequence i'_n is given by

$$i'_{mN:mN+N-1} = \sigma^m(H_0)$$

where $i'_{k(L-N/2):k(L-N/2)+L-N/2-1} = i'_{0:L-N/2-1}$ for any k.

Step 2: Insert edges from 1-factor to complete K_N.

For $m = 0, 1, \ldots, N/2 - 2$, the edges

$$(e_m, f_m) = \left(N + 2 - \sigma^{2m+3}\left(\frac{N}{2}\right), \sigma^{2m+3}\left(\frac{N}{2}\right)\right)$$

and $(N/2 + 1, 1)$ form a 1-factor and do not appear in any of the Hamiltonian cycles from step 1. To complete the graph K_N, insert edges of the 1-factor as follows. For the first Hamiltonian cycle H_0, insert two edges via

$$\begin{aligned}(i, j)_0 &= (i', j')_0\\ (i, j)_1 &= (N/2 + 1, 1)\\ (i, j)_{2:N-2} &= (i', j')_{1:N-3}\\ (i, j)_{N-1} &= (e_0, f_0)\\ (i, j)_{N:N+1} &= (i', j')_{N-2:N-1}\end{aligned}$$

where node i_n disseminates its estimate of the (i_n, j_n) channel at time n. For $m = 1, \ldots, N/2 - 2$, insert one edge into each of the other Hamiltonian cycles H_m via

$$\begin{aligned}(i, j)_{m(N+1)+1:m(N+1)+\ell} &= (i', j')_{mN:mN+\ell-1}\\ (i, j)_{m(N+1)+\ell+1} &= (e_m, f_m)\\ (i, j)_{m(N+1)+\ell+2:m(N+1)+N+1} &= (i', j')_{mN+\ell:mN+N-1}\end{aligned}$$

where $\ell = N - 2 - 2m$.

As an example, if node i_n disseminates its estimate of the (i_n, j_n) channel at time n, the length $L = 15$ periodic protocol for $N = 6$ is given by

$$\begin{aligned}i_{0:L-1} &= 1, \mathbf{4}, 2, 3, 6, \mathbf{2}, 4, 5 \mid 1, 3, \mathbf{5}, 4, 2, 5, 6\\ j_{0:L-1} &= 6, \mathbf{1}, 1, 2, 3, \mathbf{6}, 6, 4 \mid 5, 1, \mathbf{3}, 3, 4, 2, 5\end{aligned}$$

where the bold numbers indicate insertions, and we see that the freshest CSI is disseminated except at times $n = 2, 6, 11$ where the staleness of disseminated estimates is $2P$.

The following theorem establishes the efficiency of Protocol 1 and Protocol 2.

Theorem 3 (Existence of Efficient Protocols for $M = 1$): For $M = 1$ and any $N \geq 3$, there exist efficient protocols with maximum staleness $S_{max} \leq S^*_{max} + P$ and average staleness $S_{avg} \leq S^*_{avg} + 2P/3$. Moreover, Protocol 1 and Protocol 2 achieve these bounds for odd and even N, respectively.

Proof: See Appendix B. ■

Figure 4 shows the achieved staleness for Protocol 2 at each time instant when $N = 4$ and $D = 0$. We see that over one period of the protocol (i.e., for $6 \leq n \leq 11$), the instantaneous maximum staleness is equal to L in $L - N/2$ of the time periods, and is equal to $L + 1$ in $N/2$ of the time periods, thus achieving the maximum staleness bound of Theorem 3 with equality. Also, Theorem 2 gives $S^*_{avg} = 2.5$, and we see that $S_{avg} = \frac{1}{L}\sum_{n=6}^{11} S_{avg}[n] = 2.8125 \leq S^*_{avg} + 2/3$. Thus, Fig. 4 shows the protocol is efficient.

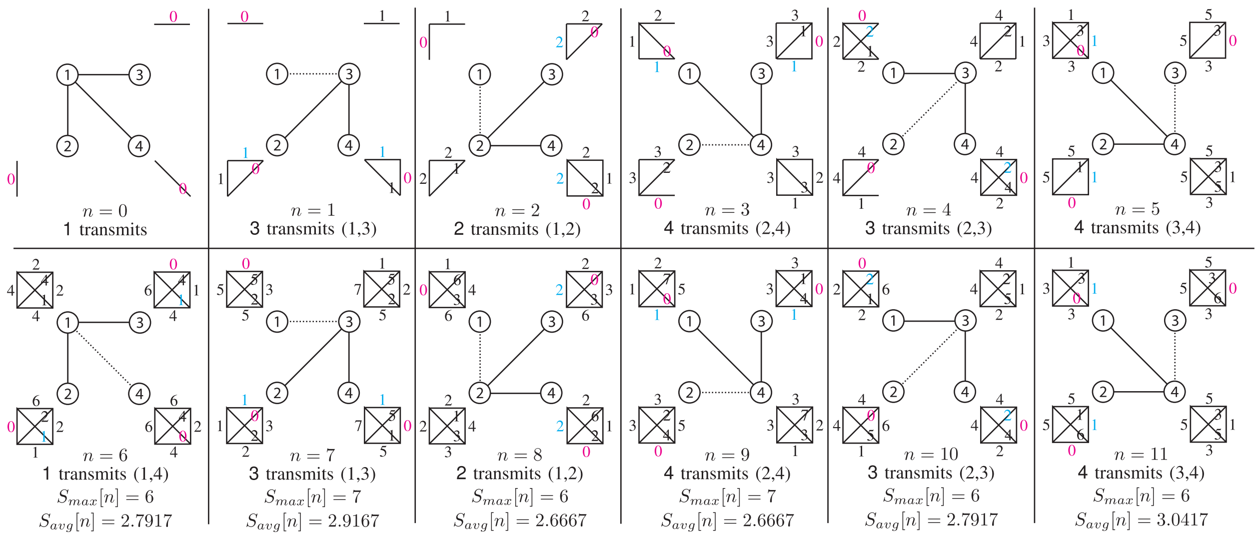

Fig. 4. Efficient CSI dissemination protocol for 4 node case. Numbers on edges indicate staleness of CSI estimates locally at each node; red numbers indicate CSI estimates have been refreshed through direct estimation, blue numbers indicate CSI refreshed through dissemination, and black numbers indicate no update to CSI since the last packet.

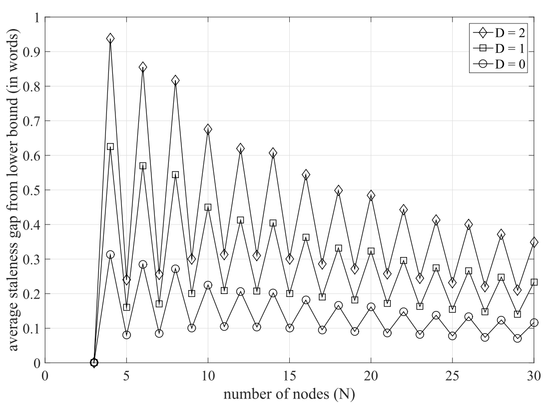

Fig. 5. Gap between the achieved average staleness of the efficient deterministic protocols with single CSI dissemination ($M = 1$) and the lower bound in Theorem 2.

Figure 5 shows a numerical example of the gap between the achieved average staleness of the efficient deterministic protocols with single CSI dissemination ($M = 1$) and the lower bound on the average staleness from Theorem 2. These results show the different behavior of the efficient deterministic protocol when N is odd or even. These results also demonstrate the $\frac{2P}{3} = \frac{2(D+1)}{3}$ achievability of the efficient deterministic protocols from Theorem 3 is conservative. In the results shown in Figure 5, the actual gaps between the achieved staleness and the lower bound appear to be better than $\frac{5(D+1)}{16}$. We conjecture the protocols from Theorem 3 are optimal in the sense that no other deterministic protocol yields better average staleness, but this requires additional combinatorial arguments beyond the scope of this work.

B. Transmitting Node Disseminates All Directly Estimated CSI

Now, we consider the other regime where in each packet the transmitting node disseminates its entire table of $N-1$ direct channel estimates. As before, we begin by presenting lower bounds on the maximum and average staleness.

Theorem 4 (Lower Bound on Maximum Staleness for $M = N-1$): For $M = N-1$, the maximum staleness S_{max} of any valid protocol is lower bounded by $S_{max} \geq S^*_{max} = (N-1)(D+N-1)$.

Proof: To arrive at this bound, we consider just the $2L$ estimates in the network that are estimated directly, and we ignore the indirect estimates. From Lemma 2 in Appendix A, it follows immediately that $S_{max} \geq S^*_{max} = (N-1)P = (N-1)(D+N-1)$. ■

Theorem 5 (Lower Bound on Average Staleness for $M = N-1$): For $M = N-1$, the average staleness S_{avg} of any valid protocol is lower bounded by $S_{avg} \geq S^*_{avg} = \frac{2N^2-2N-1}{3N}(D+N-1)$.

Proof: From Lemma 3, the $(L-N+1)N = L(N-2)$ indirect channel estimates which must be updated through dissemination have average staleness at least $S_{avg,indirect} \geq (2N-1)P/3$. From Lemma 2, the $2L$ direct channel estimates have average staleness at least $S_{avg,direct} \geq (N-1)P/2$. Hence, the weighted average then gives us a bound on the average staleness as

$$\begin{aligned} S_{avg} &= \frac{L(N-2)S_{avg,indirect} + 2LS_{avg,direct}}{LN} \\ &\geq \frac{(2N^2-2N-1)}{3N}P \end{aligned}$$

■

In this case, the efficient protocol is straightforward. There is no choice of *which* CSI to disseminate since each node always disseminates its entire table of directly estimated CSI. Once again, a periodic protocol achieves the minimum staleness bound, though as opposed to the $M = 1$ case where the period was L, the period is N in this case of $M = N-1$.

Protocol 3 ($M = N - 1$).

Node transmission order follows a round-robbin schedule, and the protocol has period N. That is, the transmitting node at time n is given by $i_n = n + 1$ for $0 \leq n \leq N - 1$, and each node disseminates its $N - 1$ direct estimates. Due to the periodicity, it follows that $i_{kN:kN+N-1} = i_{0:N-1}$ for any k.

As the following theorem shows, this N-periodic protocol is efficient.

Theorem 6 (Existence of Efficient Protocols for $M = N - 1$): For $M = N - 1$ and $N \geq 3$, there exists an efficient protocol with maximum staleness $S_{max} = S^*_{max}$ and average staleness $S_{avg} = S^*_{avg}$. Moreover, Protocol 3 achieves this bounds with equality and is therefore an efficient protocol.

Proof: All nodes transmit in round-robbin fashion so at any time n_0, all N nodes have transmitted in the last N packets. It follows immediately from Lemmas 2 and 3 that $S_{max} = S^*_{max} = (N - 1)P$ and

$$\begin{aligned} S_{avg} &= \frac{L(N-2)S_{avg,indirect} + 2LS_{avg,direct}}{LN} \\ &= \frac{2N^2 - 2N - 1}{3N} P \\ &= S^*_{avg}. \end{aligned}$$

Since the protocol achieves both the lower bounds on maximum and average staleness with equality, the deterministic round-robbin protocol is efficient for $M = N - 1$. ■

C. Extensions to the Staleness Framework

While the staleness framework developed in this paper is general, our analysis and protocols focus specifically on the case of fully-connected wireless networks with reciprocal channels, and is most applicable to small networks where dissemination of $O(N^2)$ channel gains is feasible. An interesting direction for future studies is to consider partially-connected wireless networks, i.e., incomplete (but connected) graphs. In this case, the amount of CSI to track will be less than the fully-connected scenario but some CSI may need to be disseminated over multiple hops to provide global CSI awareness to all nodes in the network, as we have shown in recent results on ring networks [46]. Related to this is the idea of encoding the disseminated CSI at higher rates, which poses an interesting tradeoff due to two competing effects: encoding at higher rates reduces transmission time which leads to more frequent CSI updates, but encoding at higher rates may reduce the connectivity of the network, requiring multiple hops to achieve global CSI. As such, the analysis of how changing the rate effects the staleness would depend heavily on the fading statistics and quality of channels between nodes.

The staleness framework developed in this paper assumes a uniform staleness weighting such that the staleness of each link is equally important in the maximum and average staleness metrics developed in Section II. In networks where the staleness of some links is more important than others, we can modify our definitions of maximum and average staleness to include non-negative staleness weights $w^{(k)}_{i,j}$, i.e.,

$$S_{max} = \max_{i,j,k,n \geq \bar{n}} w^{(k)}_{i,j} s^{(k)}_{i,j}[n] \text{ and}$$

$$S_{avg} = \frac{1}{LN} \mathrm{E}\left[\sum_{i,j,k} w^{(k)}_{i,j} s^{(k)}_{i,j}[n]\right].$$

The development of non-uniform staleness guarantees and efficient protocols for the weighted staleness metrics may be desirable when the channels have very different coherence times. Such an extension with weighted staleness metrics could also be used to model the fact that some CSI may simply be more valuable than other CSI for a given application. Further extensions to the framework might analyze yet more complex scenarios where the nodes themselves have differing, and possibly time-varying CSI requirements and priorities, or where trusted nodes are compromised and subsequently disseminate malicious CSI.

IV. Numerical Examples

This section provides a numerical example of the achieved maximum and average staleness of the efficient deterministic CSI dissemination protocols developed and analyzed in Section III, and it compares the achieved staleness with the random CSI dissemination protocols considered in [38]. These examples verify the analytical results and also allow for comparisons between the various CSI dissemination protocols in terms of the achieved maximum and average staleness. In addition, this section considers use of an alternate "greedy" protocol to assess how the staleness changes when the amount of CSI is varied between $1 \leq M \leq N - 1$. Finally, an example using the staleness bounds to assess the feasibility of global CSI in practical wireless networks is provided.

A. Comparison of Achieved Staleness and Lower Bounds

Figure 6 plots the maximum and average staleness of the efficient deterministic protocols developed in Section III and the random CSI dissemination protocols developed in [38] versus the packet data and overhead D for $N \in \{5, 25\}$. The $D = 0$ case can be considered a protocol with no data or overhead where each packet is dedicated solely to disseminating CSI. These results show that the single-CSI dissemination protocols tend be more efficient only for very small values of D, especially in the $N = 25$ case. Intuitively, when the amount of data and overhead in each packet is large, it is more efficient to disseminate $M = N - 1$ channel states in each packet since the additional staleness caused by disseminating this CSI is relatively small.

B. Staleness for Intermediate Choices of M

While the protocols presented in Section III analyze the two extremes where $M = 1$ CSI parameter is disseminated in each packet, and $M = N - 1$ CSI parameters are disseminated in each packet, it is interesting to consider the staleness

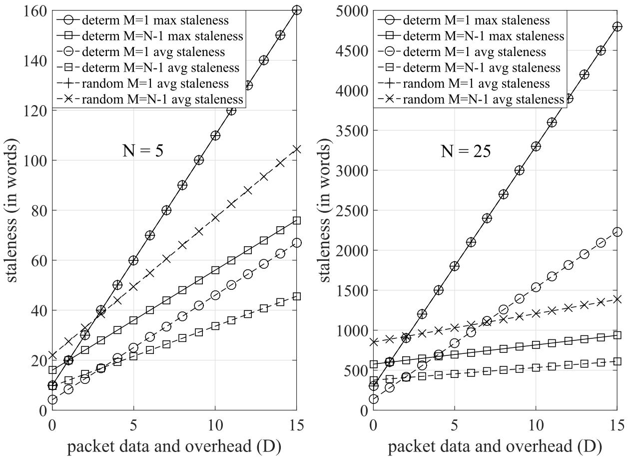

Fig. 6. Maximum and average staleness of efficient deterministic and random CSI dissemination protocols versus packet data and overhead D.

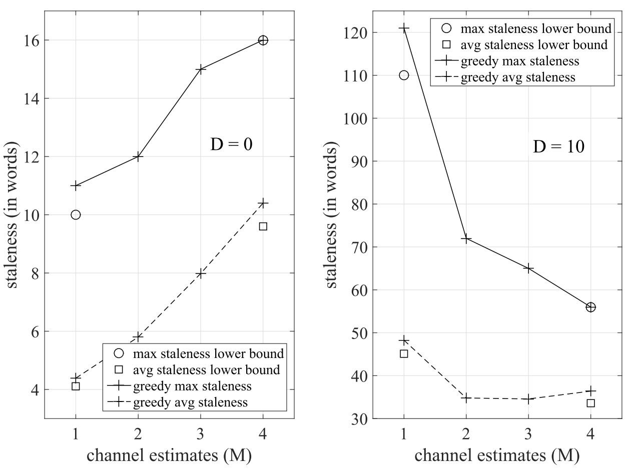

Fig. 7. Maximum and average staleness of greedy CSI dissemination protocol versus amount of CSI disseminated per packet M, for two choices of D and $N = 5$.

performance for intermediate choices of M. Derivation of bounds and efficient protocols for arbitrary M is beyond the scope of the present work, but here we present numerical results using a "greedy" protocol that, while not necessarily efficient, generates protocols for any choice of M. The greedy protocol operates by assuming, at each time instant, that an omniscient genie decides which node to transmit and which M CSI estimates should be disseminated. The genie chooses the node and CSI that minimizes *instantaneous* average staleness throughout the network. For $N = 5$ and two choices of D, Fig. 7 shows the simulated staleness as a function of M. Through experimentation, we have found that the greedy protocol is very sensitive to initialization of the staleness values throughout the network, so the results are averaged over 1000 random initial conditions. The points corresponding to the maximum and average lower bounds for $M = 1$ and $M = N - 1$ are also shown (i.e., the lower bounds from Theorems 1, 2, 4, and 5), and it is apparent that the staleness of the greedy protocol approaches these lower bounds for the chosen parameters. Again, we observe the general trend that in networks with small packet sizes, smaller values of M are preferable; on the other hand, in networks with larger packet sizes, larger values of M are preferable.

C. Comparison of Staleness With Channel Coherence Time

Finally, we consider an example applying the derived staleness bounds to a practical wireless setting. For global CSI to be useful at all nodes, the maximum staleness lower bound in seconds must be less than the coherence time of the channel. Words can be converted to seconds via

$$\text{staleness in seconds} = (\text{staleness in words}) \cdot \frac{b_{csi}}{R_b}$$

where b_{csi} is the number of bits per CSI estimate (i.e., bits per word) and R_b is the bit rate at which CSI estimates are transmitted. For a carrier frequency of f_c and an average relative speed of nodes equal to υ, the resulting Doppler spread is $f_D = \frac{\upsilon f_c}{c}$ where c is the speed of light, and the coherence time is $T_c = \frac{0.423}{f_D} = \frac{0.423c}{\upsilon f_c}$ [47]. If b_{data} bits of data and overhead are transmitted in each packet, and with each word being represented by b_{csi} bits, there are $D = b_{data}/b_{csi}$ words of data plus overhead per packet. For the maximum staleness lower bounds in Theorems 1 and 4 to be less than the coherence time, we therefore require

$$\frac{N(N-1)(b_{data}+b_{csi})}{2R_b} < \frac{0.423c}{\upsilon f_c} \quad \text{for } M = 1$$

$$\frac{(N-1)[b_{data}+(N-1)b_{csi}]}{R_b} < \frac{0.423c}{\upsilon f_c} \quad \text{for } M = N-1.$$

Consider mobile transmission of voice using LTE [48], for example, with a data rate of $R_b = 25$ Mbps, $b_{data} = 1200$ bits of data plus overhead per packet, $b_{csi} = 32$ bits per CSI estimate, a cluster of $N = 5$ fully-connected nodes, a carrier frequency of $f_c = 1900$ MHz, and $M = N - 1$ CSI estimates per packet. The bound tells us, for example, that if the speed of mobiles exceeds $\upsilon > 310$ m/s, global CSI dissemination is infeasible. Similarly, fixing $\upsilon = 100$ m/s, the bounds tell us that if the node cluster size exceeds $N > 11$, global CSI dissemination is again infeasible. The derived bounds can be used to conduct a similar feasibility analysis for communication systems with vastly different system parameters, such as 802.11 wireless local area networks.

D. Effect of Imperfect Packet Reception on Average Staleness

We consider the impact of dropped packets on the average staleness for $M = 1$. Indirectly estimated CSI acquired through dissemination requires decoding the packet contents, though CSI acquired through direct estimation can be performed without decoding the packet contents. As such, we assume that only the CSI acquired indirectly through dissemination is dropped with packet error probability p, independent over all nodes. Thus, the probability distribution of the number of disseminations needed to successfully refresh an indirectly estimated CSI parameter obeys a geometric distribution with mean $\frac{1}{1-p}$. Averaging over time gives that the average staleness of indirectly estimated CSI is $E[S_{avg,indirect}] = LP/(1-p) - P(L-1)/2$. Weighting this quantity by the fraction of

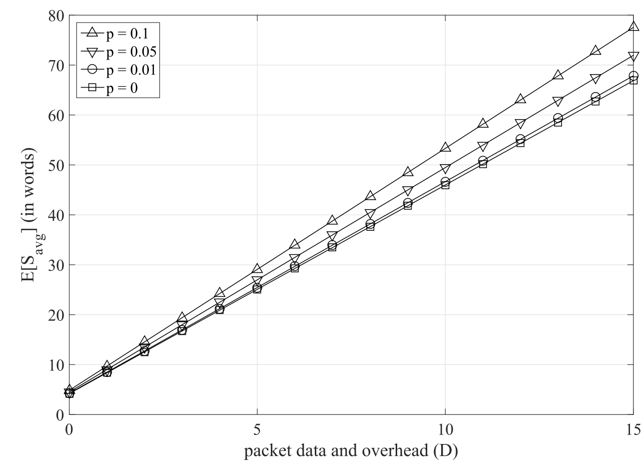

Fig. 8. Comparison of average staleness as a function of D for various packet error probabilities p, with $M = 1$, $N = 5$.

CSI that is indirectly estimated gives the total effect on average staleness due to dropped packets as

$$E[S_{avg}] = S^*_{avg} + \underbrace{\left(\frac{LPp}{1-p}\right)\left(\frac{N-2}{N}\right)}_{\substack{\text{staleness penalty}\\ \text{due to dropped packets}}} \tag{3}$$

for $M = 1$, and where S^*_{avg} is the average staleness under the assumption of perfect packet reception. This result is confirmed by simulation in Fig. 8, where indeed the simulated staleness penalty obeys the behavior described in (3).

E. Extension of Framework to Case of Non-Reciprocal Channels

An interesting extension of this framework concerns global CSI estimation and tracking for wireless networks with non-reciprocal channels, where the (i, j) channel gain is not assumed equal to the (j, i) channel gain. In such a case, the number of parameter estimates in the network doubles, and it is reasonable to expect the maximum and average staleness to also double with respect to the case of reciprocal channels. Indeed, the protocols presented in this paper for $M = 1$ can be extended for use with non-reciprocal channels by using the protocols as they are here for the first phase, and then in the second phase using a "reversed" version of the protocol where the sequence of node transmissions arises by traversing the Hamiltonian path in the opposite direction, which has the effect of doubling the protocol schedule length, and hence the staleness. The protocols for $M = N-1$ need no modification, as all directly estimated CSI is disseminated by each node; the staleness doubles in this case as well with respect to the case of reciprocal channels since each non-reciprocal CSI estimate is only updated once in each N transmissions (rather than twice for each reciprocal CSI estimate). As an example, the protocol for $N = 3$ and $M = 1$ described in Table II would become that shown in Table III. Simulations with non-reciprocal channels for all of the cases treated in Fig. 6 were repeated, and yielded an exact doubling of maximum and average staleness, though

TABLE III
3-Node Deterministic Protocol for Non-Reciprocal Channels

time	CHANNELS transmitting node	disseminated channel
$n = 0, 6, 12, \ldots$	1	(3,1)
$n = 1, 7, 13, \ldots$	2	(1,2)
$n = 2, 8, 14, \ldots$	3	(2,3)
$n = 3, 9, 15, \ldots$	2	(3,2)
$n = 4, 10, 16, \ldots$	1	(2,1)
$n = 5, 11, 17, \ldots$	3	(1,3)

we omit the figure due to lack of space and for its similarity to Fig. 6.

V. Conclusion

This paper developed a novel framework for analyzing the staleness of CSI, including staleness lower bounds, achievable upper bounds, and efficient protocols for global channel state estimation and dissemination in fully-connected wireless networks with packetized transmissions. The deterministic protocols developed in this paper were shown to be efficient in the sense that they minimize the maximum staleness across the network and also achieve a lower bound on average staleness to within a small, constant gap. The results provide engineering guidelines on the feasibility of tracking global CSI as a function of network size, the packet size and composition, packet error rate, and channel coherence time.

Appendix A
Lemmas Used in Theorem Proofs

Lemma 1: Disseminated CSI never replaces CSI that can be directly estimated.

Proof: Consider node i's estimate of the (i, j) channel, which is a channel gain that can be directly estimated by node i. The only other node that can make a direct estimate of this channel is node j. If node j disseminates its estimate of the (i, j) channel, node i would discard the indirect estimate from node j (which has non-zero staleness), and would instead form a direct estimate based on node j's transmission since it would have staleness zero. If a node other than i or j disseminates the (i, j) channel, that node must have learned the CSI indirectly from previous CSI dissemination by either node i or node j, so such a transmission is effectively re-disseminating old information. Hence, node i ignores all disseminated estimates of the (i, j) channel since the staleness of a node's directly estimated CSI can never be improved via other nodes' disseminated estimates. ■

Lemma 2: If at time $n = n_0$, all N nodes have each transmitted once in the last N packets, $S_{max,direct}[n_0] = (N-1)P$ and $S_{avg,direct}[n_0] = (N-1)P/2$. Furthermore, if at time $n = n_0$ one or more of the N nodes have not transmitted in the last N packets, $S_{max,direct}[n_0] > (N-1)P$ and $S_{avg,direct}[n_0] > (N-1)P/2$.

Proof: From Lemma 1, we can ignore the effect of dissemination on directly estimated CSI. Transmission of a single packet generates $N-1$ fresh, direct estimates throughout the network. Thus, if the N nodes have each transmitted in the last N packets, there are $N-1$ estimates with staleness 0, $N-1$ with staleness P, $N-1$ with staleness $2P$, ..., and $N-1$ with staleness $(N-1)P$. It then follows trivially that the maximum staleness is $(N-1)P$ and the average is $(N-1)P/2$. If any node has *not* transmitted in the last N packets, there must be at least one group of $N-1$ direct estimates with staleness greater than $(N-1)P$, resulting in a larger maximum and a larger average. ■

Lemma 3: Let $M = N - 1$. If at time $n = n_0$, all N nodes have each transmitted once in the last N packets, $S_{max,indirect}[n_0] = (N-1)P$ and $S_{avg,indirect}[n_0] = (2N-1)P/3$. Furthermore, if at time $n = n_0$ one or more of the N nodes have not transmitted in the last N packets, $S_{max,indirect}[n_0] > (N-1)P$ and $S_{avg,indirect}[n_0] > (2N-1)P/3$.

Proof: Assume without loss of generality that nodes $1, 2, \dots, N$ transmit at times $n = n_0 - N + 1, n_0 - N + 2, \dots, n_0$, respectively, so at time $n = n_0$ each of the $N-1$ nodes have transmitted once in the previous $N-1$ packets. At time $n = n_0$, node N's table of $N-1$ direct channel estimates has staleness $P, 2P, \dots, (N-1)P$ since these direct estimates were made during the transmissions of each of the other $(N-1)$ nodes. Recall that a single disseminated estimate is only used by $N-2$ receiving nodes since, from Lemma 1, nodes which can make direct estimates of a given channel ignore disseminated estimates. Thus, node's N transmission at time n_0 refreshes the indirect estimates throughout the network so there are $N-2$ indirect estimates with staleness P, $N-2$ indirect estimates with staleness $2P, \dots, N-2$ indirect estimates with staleness $(N-1)P$.

Consider node i's table of directly estimated CSI at time $n = n_0-N+i$. Since node i's direct estimates of channels between itself and nodes $i+1, i+2, \dots, N$ will be overwritten by later, fresher direct estimates in subsequent packets at times $n_0-N+i+1 < n \le n_0$, consider just the $(i-1)$ direct estimates between node i and nodes $1, 2, \dots, i-1$ that will not be refreshed or overwritten by time $n = n_0$. When node i disseminates its direct channel estimates at time $n = n_0-N+i$, its transmission refreshes the indirect estimates throughout the network so (of those that will not be overwritten) there are $N-2$ indirect estimates with staleness P, $N-2$ indirect estimates with staleness $2P, \dots, N-2$ indirect estimates with staleness $(i-1)P$. By time $n = n_0$, these estimates are still present throughout the network since no fresher estimates of these $(i-1)$ channels have been disseminated; however, they have grown staler by $(N-i)$ packets so there are $N-2$ indirect estimates with staleness $(N-i+1)P$, $N-2$ indirect estimates with staleness $(N-i+2)P, \dots, N-2$ indirect estimates with staleness $(N-1)P$.

Summing for i over all N nodes, then, we see that at time $n = n_0$, the whole network has $1(N-2)$ indirect estimates with staleness $1P$, $2(N-2)$ indirect estimates with staleness $2P, \dots, (N-1)(N-2)$ indirect estimates with staleness $(N-1)P$. The average staleness of these $L(N-2)$ indirect estimates is then

$$\frac{1}{L(N-2)} \sum_{k=1}^{N-1} k(N-2)kP = \frac{P}{L} \sum_{k=1}^{N-1} k^2 = \frac{(2N-1)P}{3}.$$

If at least one node has transmitted more than once during the last N transmissions so at least one node has *not* transmitted in the last N transmission, then there will be at least one channel estimate with staleness greater than $(N-1)P$. To see this, assume that node i does not transmit in any of the most recent N packets. In this case, node j's estimate of the (i, j) channel will be greater than $(N-1)P$ for any j. In addition, with a single stale direct estimate of the (i, j), the indirect estimates of the (i, j) channel throughout the network will be yet more stale, resulting in a larger average staleness. ■

APPENDIX B
PROOF OF THEOREM 3

Proof: Again, due to the lack of Eulerian tours on K_N when N is even, we have different protocols for the case of N odd and even. First, we prove the theorem for the case when N is odd, and then the case of N even. In each case, we will prove achievability of maximum staleness first, and then the achievability of average staleness.

Achievable Maximum Staleness of Protocol 1 for Odd N: First, note that an Eulerian tour on K_N exists in this case. Since an Eulerian tour is a walk, a node always disseminates the direct estimate it formed during the previous packet, which has staleness P. Since all edges in an Eulerian tour are visited exactly once, all channel gains are disseminated exactly once per tour, in L packets. In addition, each vertex is visited $(N-1)/2$ times in an Eulerian tour of K_N, so each node transmits $(N-1)/2$ times, and therefore both conditions for valid protocols are satisfied. After a sequence of L transmissions corresponding to one Eulerian tour of K_N, the stalest CSI in the network (having staleness LP) will be indirect estimates of the channel corresponding to the first edge traversed in the Eulerian tour. Since an Eulerian tour starts and ends at the same vertex, the node corresponding to the starting vertex has a fresh estimate of the stalest channel, and it must be disseminated in the next time slot to maintain minimum maximum staleness LP. Thus, if the Eulerian tour repeats, the instantaneous maximum staleness remains constant at $S_{max}[n] = LP$, and therefore $S_{max} = S^*_{max}$.

Achievable Average Staleness of Protocol 1 for Odd N: The sequence of transmitting nodes in Protocol 1 is exactly the "zig-zag" Hamiltonian cycle decomposition of K_N from the Lucas-Walecki construction [44], [45], and it is L-periodic since it is a repetitive Eulerian tour. Consider just the $L(N-2)$ indirect channel estimates which must be updated through dissemination. Since the protocol corresponds to an Eulerian tour, at any time $n = n_0$ each of the L distinct channel gains were disseminated in the last L packets. Since each node always disseminates the freshest information, at any time $n = n_0$ there are $N-2$ indirect estimates with staleness P, $N-2$ of them with staleness $2P, \dots$, and $N-2$ of them with staleness LP. Thus, the average staleness of the indirectly estimated CSI is always $S_{avg,indirect}[n] = (L+1)P/2$.

Next, consider the $2L$ channel estimates which can be directly estimated. Because the protocol consists of Hamiltonian cycle decompositions, at every time $n = n_0$ that is a multiple of N, the last N packets were all transmitted by distinct nodes. Thus, at times which are a multiple of N, Lemma 2 applies, and the average staleness of the directly estimated CSI is $(N-1)P/2$. However, for times that are *not* a multiple of N, the last N packets cross a N-block boundary, and the average staleness will be higher since the last N packets are not necessarily transmitted by all N distinct nodes. However, at time $n = mN+k$ for appropriate integers m and $1 \leq k \leq N-1$, the last k packets were transmitted by distinct nodes. In the worst case, the previous k packets before $n = mN$ from the previous N-block were transmitted by the same k nodes, which provides an upper bound on the average staleness of the directly estimated CSI. Thus, the average staleness of the k direct estimates made in the current N-block is $(k-1)/2$, and the average staleness of the $N-k$ direct estimates made during the previous N-block is upper bounded by $(2k+N+k-1)/2$, giving

$$S_{avg,direct}[mN+k] \leq \frac{k\frac{k-1}{2} + (N-k)\frac{N+3k-1}{2}}{N} P$$

where the first term in the numerator represents the weighted average staleness of the k direct estimates in the current N-block, and the second term represents the average staleness of the $N-k$ direct estimates made during the previous N-block. This expression achieves its maximum when $k = N/2$, and results in average staleness of the directly estimated CSI as

$$S_{avg,direct}[n] \leq \frac{3N-2}{4} P \tag{4}$$

for all n. Finally, the weighted average gives us an upper bound on the average staleness of this protocol as

$$\begin{aligned} S_{avg} &= \frac{L(N-2)S_{avg,indirect} + 2LS_{avg,direct}}{LN} \\ &\leq \frac{(N^3 - 3N^2 + 10N - 8)}{4N} P \\ &= S^*_{avg} + \frac{P}{2}. \end{aligned}$$

Since the maximum staleness of Protocol 1 is equal S^*_{max}, and the average staleness is within a constant gap of the lower bound S^*_{avg}, the protocol is efficient for odd N.

Achievable Maximum Staleness of Protocol 2 for Even N: The sequence i'_n of transmitting nodes developed in step 1 of Protocol 2 is exactly the "zig-zag" Hamiltonian cycle decomposition of $K_N - I$ from the Lucas-Walecki construction [44], [45]. Thus, in addition to being an Eulerian tour of $K_N - I$, the construction guarantees every node is represented exactly once in each block of N in the sequence i'_n.

Note that in step 2, the missing edges of the 1-factor are exactly those given in [45]. Furthermore, the two missing edges $(N/2+1, 1)$ and $(N/2-1, N/2+3)$ inserted in the first Hamiltonian cycle at times $n_0 = 1$ and $n_1 = N-1$, respectively, obey the property $i_{n-1} = j_n = j_{n+1}$ for $n = n_0$ or $n = n_1$. This follows from the definition of the zig-zag H_0, which determines i_{n-1} and j_{n+1} at $n = n_0$ and $n = n_1$. Insertion of these two edges therefore results in dissemination of CSI that is $2P$ packets old at times n_0+1 and n_1+1 since nodes i_{n_0+1} and i_{n_1+1} disseminate estimates formed at times n_0-1 and n_1-1, respectively. The disseminated estimates at all other times in the first Hamiltonian cycle have staleness P. Thus, insertion of the two missing edges in the creation of (i_n, j_n) extends the first N-block of i'_n to have length $N+2$, with N estimates being disseminated with staleness P, and 2 estimates being disseminated with staleness $2P$.

Using similar arguments, insertion of the remaining missing edges in subsequent Hamiltonian cycles at times $n = m(N+1)+\ell+1$ for $m = 1, \ldots, N/2-2$ extends each of the remaining N-blocks of i'_n to have length $N+1$, with N estimates being disseminated with staleness P, and 1 estimate being disseminated with staleness $2P$. Summarizing, the period-L protocol consists of one block of $N+2$ disseminated estimates followed by $N/2-2$ blocks of $N+1$ disseminated estimates. Due to use of the Lucas-Walecki construction, each of the N nodes transmits at least once in each block, with 2 nodes transmitting twice in the first block, and 1 node transmitting twice in subsequent blocks. Furthermore, the period L sequence disseminates $L - N/2$ estimates with staleness P, and $N/2$ estimates with staleness $2P$. After $L-1$ packets, a disseminated estimate with staleness $2P$ will have staleness $(L-1)P + 2P = LP + P$, so $S_{max} = S^*_{max} + P$.

Achievable Average Staleness of Protocol 2 for Even N: There are $L - N/2$ indirect estimates with average staleness $(L+1)P/2$ (i.e., those which are always disseminated with staleness P), and $N/2$ indirect estimates with average staleness $(L+1)P/2 + P$ (those which are disseminated with staleness $2P$), giving

$$\begin{aligned} S_{avg,indirect} &= \frac{(L - \frac{N}{2})\frac{(L+1)P}{2} + \frac{N}{2}(\frac{(L+1)P}{2} + P)}{L} \\ &= \frac{(L+1)P}{2} + \frac{N}{2L} P. \end{aligned}$$

Next, consider the $2L$ channel estimates which can be directly estimated. The instantaneous average staleness is largest when the most recent N packets cross a Hamiltonian cycle block boundary so the last N transmissions are not all from distinct nodes. Using arguments identical to those in arriving at equation (4), the average staleness of the directly estimated CSI is upper bounded by

$$S_{avg,direct} \leq \frac{3N-2}{4} P.$$

Finally, the weighted average gives us an upper bound on the average staleness of this protocol as

$$\begin{aligned} S_{avg} &= \frac{L(N-2)S_{avg,indirect} + 2LS_{avg,direct}}{LN} \\ &\leq \frac{(N^3 - 4N^2 + 13N - 14)}{4(N-1)} P \\ &= S^*_{avg} + \frac{N^2 + N - 4}{2N^2 - 2N} P \\ &\leq S^*_{avg} + \frac{2P}{3} \end{aligned}$$

for $N \geq 4$. Since the maximum and average staleness of Protocol 2 is within a constant gap of the bounds S^*_{max} and S^*_{avg}, the protocol is efficient for even N. ■

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