

A CHEAP RELAYING PROTOCOL FOR ORTHOGONAL RELAY CHANNELS

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ABSTRACT

The proposed relaying scheme, which is an optimized scalar quantize-and-forward (QF) protocol, has at least three attractive features: 1. it is simple; 2. it exploits the signal-to-noise ratios (SNR) of the source-relay and relay-destination channels; 3. it can be seen as a digital alternative of the conventional (analog) amplify-and-forward (AF) in a digital relay transceiver. The presented QF protocol is optimized in terms of end-to-end distortion, extending the idea of joint source-channel coding. Using this cooperation protocol over orthogonal relay channels, it is shown that the quantization noise introduced by the relay can significantly degrade the receiver performance if the latter uses a maximum ratio combiner (MRC) to combine the two signals from the source and relay. In order for the receiver to compensate for this effect, we propose a maximum likelihood detector (MLD), which is optimum for the QF protocol.

Index Terms— Relay channel, quantize-and-forward, joint source-channel coding, ML detector, amplify-and-forward.

1. INTRODUCTION

Considered relay channels are characterized by the fact that the source-destination and relay-destination links are assumed to be orthogonal. For the channels under investigation there are at least two important technical issues: the relaying protocol and the signal recombination scheme at the destination. Three main types of relaying protocols have been considered in the literature: amplify-and-forward, decode-and-forward (DF) and estimate-and-forward (EF). From the corresponding works, several observations can be made: (a) from information-theoretic studies like [1][2] it appears that the best choice of the relaying scheme depends on the source-relay channel (i.e. the backward channel) SNR and that of the relay-destination channel; (b) there are not many works dedicated to the design of practical EF schemes although the EF protocol has the potential to perform well for a wide range of relay receive SNRs (in contrast with DF which is generally more suited to relatively high SNRs).

One of the motivations for the work presented in the paper is precisely to propose a low-complexity relaying scheme (comparable to the AF protocol complexity) that can be implemented in a *digital* relay transceiver (in contrast with the AF protocol) and use the knowledge of the SNRs of the forward (source-relay) and backward (relay-destination) channels in order for the relay to optimally adapt to the forward and backward channel conditions. To achieve these goals, the main solution proposed is a QF protocol for which forwarding is done on a symbol-by-symbol basis and aims to minimize the mean square error (MSE) between the source signal and its reconstructed

version at the output of the dequantizer at the destination. Some researchers have also referred to the classic Wyner-Ziv source coding scheme in [3] as QF [4][5]. Our practical approach, which ultimately aims to minimize the raw bit error rate (BER) at the destination for a fixed transmit spectral efficiency and does not exploit error correcting coding, differs from these information-theoretic works. It also differs from other practical studies on EF protocols, such as [6] and [7] where the authors consider the non-orthogonal half-duplex relay channel and focus on the achievable rate of the designed EF protocol. The authors also note the different approach of [8] where the Wyner-Ziv idea is used to quantize the decoder soft output at the relay and the source-destination signal is used as a side information at the receiver. At last, the schemes such as those presented in [6][7][8] are not analytically optimized by taking the SNRs of the backward and forward channels into account. Rather, our work is based on the joint source-channel coding approach originally introduced in [9] for the Gaussian point-to-point channel where the authors extended the original iterative Lloyd's algorithm by designing a scalar quantizer that takes into account the channel through which the quantized Gaussian source is to be transmitted. In this paper we further extend the iterative algorithm of [9] in the context of quasi-static orthogonal relay channels by taking into account both the forward and backward channels and providing a non-restrictive sufficient condition for convergence of the derived algorithm, similarly to [10].

This paper is organized as follows: in Sec. 2 the signal model, main assumptions, and notation are given. In Sec. 3 the proposed QF scheme and underlying ML detector are provided. In Sec. 4 the proposed scheme is evaluated in terms of raw BER and compared with AF. Concluding remarks are provided in Sec. 5.

2. SYSTEM MODEL

The source is assumed to be represented by a discrete-time unit-power signal x ($E[|x^2|] = 1$), which takes its value in the finite set of equiprobable symbols $\mathcal{X} = \{x_1, \dots, x_{M_s}\}$. For sake of simplicity, a square M_s -QAM constellation is assumed. More importantly, the samples of the source, denoted by $x(n)$ where n is the time index, are assumed to be independent and identically distributed (i.i.d.) as in [9][10]. In the context of digital communications this assumption is generally valid because of interleaving, dithering or equivalent operations. In order to limit the relay and receiver complexity we will not exploit the interactions between the quantizer and the error correcting coders, possibly present at the source and relay. Therefore the assumption made on the source samples and channel model (described just below) implies that there is loss of optimality by assuming *scalar* quantizers, i.e. symbol-by-symbol forwarding at the relay, instead of vector quantizers. At each time instant n the source broad-

casts the signal $x(n)$, which is received by the destination and relay nodes. The received baseband signals can be written: $y_{sd}(n) = h_{sd}x(n) + w_{sd}(n)$, $x_{sr}(n) = h_{sr}x(n) + w_{sr}(n)$ where w_{sd} and w_{sr} are zero-mean circularly symmetric complex Gaussian noises with variances σ_{sd}^2 and σ_{sr}^2 respectively. The complex coefficients h_{sd} and h_{sr} are, respectively, the gains of the source-destination and source-relay channels. In this paper, for simplicity of presentation, most of the derivations are conducted for static channels, so h_{sd} and h_{sr} are constant over the whole transmission. However, all the results provided easily extend to quasi-static channels, in which case these quantities are constant over a block duration and vary from block to block. In the simulation part both cases will be analyzed and Rayleigh block-fading will be assumed for modeling the channel gains in the case of quasi-static channels. The relay forwards the cooperation signal $x_r(n)$ to the destination. We assume memoryless and zero-delay relaying. Under these assumptions, $x_r(n)$, which satisfies the average power constraint $E[|x_r|^2] = 1$, is the result of a zero-memory quantization operation (denoted by \mathcal{Q}) on the sample $x_{sr}(n)$ followed by an M_r -QAM modulation (denoted by \mathcal{M}). Since the relay function and channels are memoryless, in the sequel we will at times omit the time index n from the signals. The cooperation signal received at the destination is written $y_{rd}(n) = h_{rd}x_r(n) + w_{rd}(n)$. Orthogonality between the received cooperation signal y_{rd} and direct signal y_{sd} can be implemented by frequency division (FD) and we assume that y_{sd} and y_{rd} have the same bandwidth.

At the destination, two types of combiners can be assumed. We will use either a conventional MRC or a more sophisticated detector, namely the MLD, which will be derived in Sec. 3.2. Fig. 1 summarizes the system model. The notation \mathcal{D} stands for decoder, which jointly incorporates the demodulation and de-quantization operations.

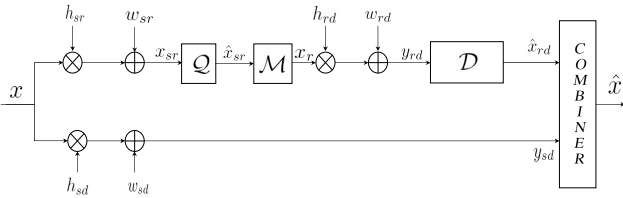


Fig. 1. System model

3. QUANTIZE-AND-FORWARD

3.1. Relaying protocol description

The most natural way to estimate and forward the signal received by the relay is to quantize x_{sr} in order to minimize the distortion $D_{00} = E[|\hat{x}_{sr} - x_{sr}|^2]$, map the quantizer output onto a QAM modulation and send it to the destination. In the high cooperation regime (i.e. $\frac{1}{\sigma_{rd}^2} \gg 1$) this strategy is almost optimal since it almost achieves the performance of a 1×2 single input multiple output (SIMO) system. On the other hand if x_{sr} is quantized with a reasonably high number of bits and sent through a bad cooperation channel, minimizing D_{00} is no longer optimal. This is why minimizing $D_{01} = E[|\hat{x}_{rd} - x_{sr}|^2]$ can be more efficient as shown by [9][10][11][12] in the context of the point-to-point Gaussian channel. In the context of the relay channel we know that the source-relay channel quality also plays a role in the receiver performance. Therefore we propose to minimize the MSE between

the reconstructed signal \hat{x}_{rd} and the original source signal x i.e. $D_{11} = E[|\hat{x}_{rd} - x|^2]$ by assuming the SNRs of the forward and backward channels known to the relay.

Let us turn our attention to the quantizer itself. Since the signal to be quantized is complex, the quantizer is made of two “sub-quantizers” for the real and imaginary parts of x_{sr} . The quantization consists in mapping the signal x_{sr} into a pair of rational numbers belonging to $\mathcal{V}^R \times \mathcal{V}^I = \{v_1^R, v_2^R, \dots, v_L^R\} \times \{v_1^I, v_2^I, \dots, v_L^I\}$ where $L = 2^{\frac{b}{2}}$ and b is the total number of quantization bits. Note that the real and imaginary parts of the signal received by the relay are generally independent in practice, which allows us to design them independently. As a QAM modulation is assumed at the source we can restrict our attention to the sub-quantizer Q^R for the real part of x_{sr} . The sub-quantizer maps $\text{Re}(x_{sr}) = x_{sr}^R$ onto the finite set $\{v_1^R, v_2^R, \dots, v_L^R\}$. The mapping is done as follows: if $x_{sr}^R \in S_j^R$ then $Q^R(x_{sr}^R) = v_j^R$ where $S_j^R = [u_j^R, u_{j+1}^R)$ for all $j \in \{1, 2, \dots, L\}$ and $\{u_j\}_{j \in \{1, \dots, L\}}$ are called the transition levels. We will denote $\mathcal{U}^R = \{u_1^R, u_2^R, \dots, u_{L+1}^R\}$. The same procedure is applied to the signal $x_{sr}^I = \text{Im}(x_{sr})$. The quantizer output is then mapped onto the constellation. The quantizer output is then mapped onto the constellation following the idea of [13]. The mapping is done in such a manner that close representatives in the signal space are assigned to close symbols in the modulation space. Therefore, the most likely decision errors which appear in the neighborhood of the symbol associated with the input representative will result in a slight increase in distortion.

Let us focus now on the quantizer optimization procedure. To find the optimal pair of sub-quantizers at the relay we minimize D_{11} as follows. The distortion can be written as:

$$D_{11} = \underbrace{E[(\hat{x}_{rd}^R)^2] - 2E[\hat{x}_{rd}^R x^R] + E[(x^R)^2]}_{D_{11}^R} + \underbrace{E[(\hat{x}_{rd}^I)^2] - 2E[\hat{x}_{rd}^I x^I] + E[(x^I)^2]}_{D_{11}^I}. \quad (1)$$

As D_{11}^R and D_{11}^I can be optimized independently and identically we focus, hence forth, on minimizing D_{11}^R . Given a number of quantization bits we now optimize the sub-quantizer Q^R by minimizing D_{11}^R with respect to the transition levels $\{u_\ell\}_{\ell \in \{1, \dots, L\}}$ and the representatives $\{v_\ell\}_{\ell \in \{1, \dots, L\}}$. For fixed transition levels the optimum representatives are the centroids of the corresponding quantization cells which are obtained by setting the partial derivatives of D_{11}^R to zero:

$$v_\ell^R = \frac{\sum_{k=1}^{\sqrt{M_s}} x_k^R p_k \sum_{j=1}^L P_{j,\ell}^R \int_{u_j^R}^{u_{j+1}^R} \phi(t - x_k^R) dt}{\sum_{k=1}^{\sqrt{M_s}} p_k \sum_{j=1}^L P_{j,\ell}^R \int_{u_j^R}^{u_{j+1}^R} \phi(t - x_k^R) dt}. \quad (2)$$

where $\forall k \in \{1, \dots, \sqrt{M_s}\}$, $p_k = \Pr[X^R = x_k^R]$ (i.e. the input statistics), $\forall (j, \ell) \in \{1, \dots, L\}^2$, $P_{j,\ell}^R = \Pr[\hat{x}_{rd}^R = v_\ell^R | \hat{x}_{sr}^R = v_j^R]$ (i.e. the forward channel statistics) and $\phi(t) = \frac{|h_{sr}|}{\sqrt{\pi\sigma_{sr}}} e^{-\frac{|h_{sr}|^2 t^2}{\sigma_{sr}^2}}$ is the Gaussian probability density function of the real noise component $\text{Re}(w_{sr})$ of the signal received by the relay (i.e. the backward channel statistics). When the representatives are fixed it is not trivial, in general, to determine the transition levels explicitly as is the

case of conventional channel optimized quantizers such as [10] for which the backward channel is not present. Determining the transition levels then requires the use of an exhaustive search algorithm. However, note that there are simple cases such as the 4-QAM at the source, which is used in the simulations in Section 4, where both the optimum representatives for fixed transition levels and optimum transition levels for fixed representatives can be found. For a 4-QAM constellation we have $(x^R, x^I) \in \{-A, +A\}^2$. For fixed transition levels, the representatives are obtained by replacing x_k^R by its values in (2). And, for fixed representatives we have

$$u_\ell^{R,*} = \frac{\sigma_{sr}^2}{2A} \ln \left[\frac{\sum_{k=1}^L (P_{\ell,k}^R - P_{\ell-1,k}^R) \left(A + \frac{1}{2} v_k^R \right) v_k^R}{\sum_{k=1}^L (P_{\ell,k}^R - P_{\ell-1,k}^R) \left(A - \frac{1}{2} v_k^R \right) v_k^R} \right]. \quad (3)$$

Note that in (3) the strict positiveness of the argument of the natural logarithm insures the existence of the optimum transition levels. We are now in position to provide the complete iterative optimization procedure. Let i and ϵ be the iteration index and the current value of the estimation error criterion of the iterative algorithm. The algorithm is said to have converged when ϵ reaches ϵ_{max} . **Step 1:** Set $i = 0$. Set $\epsilon = 1$. Initialize \mathcal{V}^R and \mathcal{U}^R with the sets (say $\mathcal{V}_{(0)}^R$ and $\mathcal{U}_{(0)}^R$) obtained from the algorithm in [10], which corresponds to a local optimum since the backward channel is not taken into account. **Step 2:** Set $i \rightarrow i + 1$. For the fixed partition $\mathcal{U}_{(i-1)}^R$ use equation (2) to find the optimal codebook $\mathcal{V}_{(i)}^R$. For the fixed codebook $\mathcal{V}_{(i)}^R$ use equation (3) to obtain the optimal partition $\mathcal{U}_{(i)}^R$. If the realizability condition $u_1^R \leq u_2^R \leq \dots \leq u_L^R$ is not met stop the procedure and keep the transition levels provided by the previous iteration. **Step 3:** Up-

date ϵ as follows: $\epsilon = \frac{\sum_{k=1}^L |v_{k(i)}^R - v_{k(i-1)}^R|}{\sum_{k=1}^L |v_{k(i)}^R|}$. If $\epsilon \geq \epsilon_{max}$ then go to Step 2; Stop otherwise.

As with other iterative algorithms (e.g. the EM algorithm) one cannot easily prove or insure, in general, the convergence to the global optimum. When the backward channel is not present the authors of [10] proved that the distortion obtained by applying the generalized Lloyd's algorithm is a non-increasing function of i and provide a sufficient condition under which the procedure is guaranteed to converge towards a local optimum. The corresponding condition is not restrictive since it can be imposed through the realizability constraint (u_ℓ must be an increasing function of ℓ) of the transition levels [10] to the iterative procedure without loss of optimality. It turns out a similar result can be derived in our context if one assumes a zero-mean channel input (i.e. $E[X^R] = 0$) and the backward channel to be an AWGN channel. This condition can be proved to be: $\forall \ell \in \{1, \dots, L-1\}$, $E[\hat{X}_{rd}^R | \hat{X}_{sr}^R = v_{\ell+1}^R] > E[\hat{X}_{rd}^R | \hat{X}_{sr}^R = v_\ell^R]$. If this condition is met the MSE will be a non-increasing function of the iteration index. Because of the lack of space the proof of this result is omitted.

3.2. Maximum likelihood detector for the QF protocol

As mentioned in section 2 the purpose of the combiner is to combine the source-destination signal y_{sd} and the dequantizer output \hat{x}_{rd} . If one decomposes the latter signal as $\hat{X}_{rd} = X + \hat{W}_{rd}$ it is obvious that the noise component \hat{W}_{rd} is correlated with the

useful signal component and is not Gaussian in general. Therefore maximizing the output SNR of a linear combiner is not optimum in terms of raw BER. In order to extract the best of the cooperation between the receiver and relay for all channel SNRs we propose to use a non-linear combiner namely the ML detector. Assume that the symbol transmitted by the source is x and the $Q(x_{sr}) = v_i$. The likelihood $p_{ML} = p(y_{sd}, \hat{x}_{rd} | x)$ can be shown to factorize as:

$p_{ML} = p(y_{sd} | x) p(\hat{x}_{rd} | x)$ where $p(y_{sd} | x) = \frac{1}{\pi \sigma_{sd}^2} e^{-\frac{|y_{sd} - h_{sd} x|^2}{\sigma_{sd}^2}}$. For expanding the second term $p(\hat{x}_{rd} | x)$ one has to remind that $\hat{X}_{rd} \in \mathcal{V}^R \times \mathcal{V}^I = \{v_1, v_2, \dots, v_{M_r}\}$ and makes use of the channel transitions probabilities $P_{k,\ell}$ between complex representatives (see section 3.1 where we have defined $P_{k,\ell}^R$ for the real part of complex representatives). We have:

$$\begin{aligned} p(\hat{x}_{rd} = v_i | x) &= \int_{x_{sr}} p(x_{sr}, \hat{x}_{rd} = v_i | x) dx_{sr} \\ &= \sum_{j=1}^{M_r} \left[\int_{x_{sr} \in S_j} p(x_{sr} | x) p(\hat{x}_{rd} = v_i | x_{sr}) dx_{sr} \right] \\ &= \sum_{\ell=1}^{\sqrt{(M_r)}} \sum_{m=1}^{\sqrt{(M_r)}} P_{j,i} \int_{u_\ell^R}^{u_{\ell+1}^R} \phi(t - x^R) dt \int_{u_m^I}^{u_{m+1}^I} \phi(t' - x^I) dt' \end{aligned} \quad (4)$$

where the index j corresponds to the symbol of the relay alphabet (i.e. $\{1, \dots, M_r\}$) associated with the pair of representatives (v_ℓ^R, v_m^I) . Now, by denoting $\underline{s} = (s_1, \dots, s_N)$ the vector of bits associated with the source symbol x allows us to express the log-likelihood ratio for the n^{th} bit:

$$\lambda(s_n) = \log \left[\frac{\sum_{\underline{s} \in S_1^{(n)}} p(y_{sd} | x) p(\hat{x}_{rd} | x)}{\sum_{\underline{s} \in S_0^{(n)}} p(y_{sd} | x) p(\hat{x}_{rd} | x)} \right] \quad (5)$$

where the sets $S_1^{(i)}$ and $S_0^{(i)}$ are defined by: $S_1^{(n)} = \{(s_1, \dots, s_N) \in \{0, 1\}^N | s_n = 1\}$ et $S_0^{(n)} = \{(s_1, \dots, s_N) \in \{0, 1\}^N | s_n = 0\}$. If $\lambda(s_n) > 0$ then $\hat{s}_n = 1$ and $\hat{s}_n = 0$ otherwise.

4. SIMULATION ANALYSIS

We assume a 4-QAM at the source and focus on the raw BER versus $SNR_{sr} = \frac{1}{\sigma_{sr}^2}$. Because of the lack of space we restricted our attention to a few scenarios but will also briefly comment simulations that cannot be provided here. For static channels (or quasi-static channels with a strong Rician component), Fig. 2 compares the optimum QF with the conventional AF in a typical scenario where $SNR_{sr} = SNR_{sd} + 10$ dB, $SNR_{rd} = 10$ dB and the number of quantization bits is 6 (i.e. $\frac{b}{2}$ bits per sub-quantizer). At the destination, the MRC is used for all relaying schemes. The QF solution provides a significant gain over the AF protocol. Also for making a fairer comparison, we also represented the performance of a version of the AF protocol where x_{sr} is optimally clipped to minimize the end-to-end distortion (CF), which also exploits the source-relay and relay-destination channels SNRs. For quasi-static Rayleigh fading channels, many simulations showed that the receiver performs quite similarly no matter which relaying protocol (AF, clipped AF or QF) is used, provided that the optimum combining scheme is employed (i.e. the MRC is used for AF and MLD is used for QF, see Fig. 3). This is essentially due to the averaging effect of the channel conditions. The two main points to be mentioned here are as follows: (a)

since the AF and QF protocol perform quite similarly over block fading channels, the QF protocol can be seen as a digital alternative to the AF protocol for digital relay transceivers; (b) other simulations showed that if an MRC is used in conjunction with the QF protocol the receiver performance can be degraded (see Fig. 4).

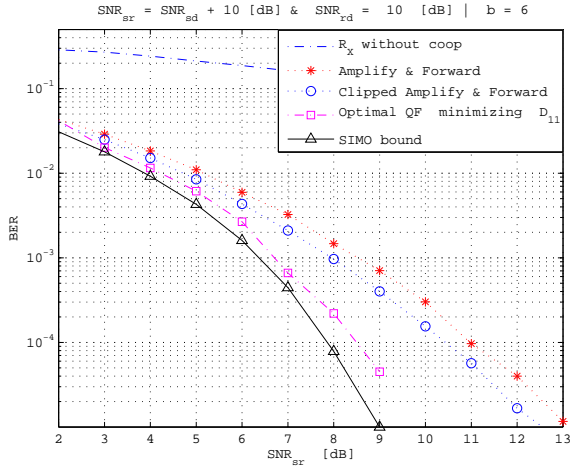


Fig. 2. Case of static channels: QF versus AF and clipped AF

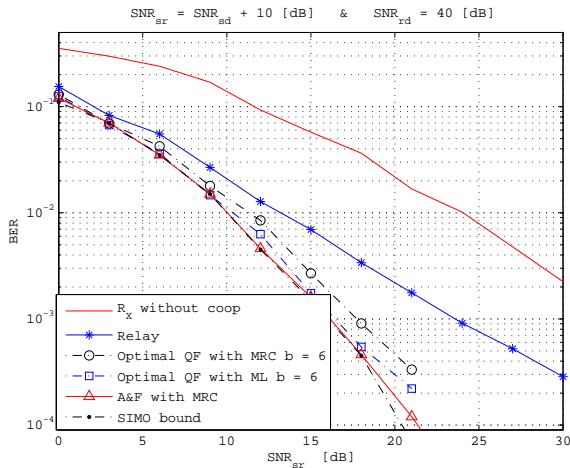


Fig. 3. Case of quasi-static channels: QF versus AF

5. CONCLUSION

The proposed low-complexity quantize-and-forward scheme generally performs close to or is better than the conventional AF protocol over channels with a strong Rician component. Over Rayleigh channels we have seen that the optimum QF protocol, provided it is associated with an ML detector, has generally similar performance to the conventional AF protocol. Since the optimum QF protocol is both scalar, simple and generally performs closely to the AF protocol, this shows that the proposed solution can be seen as a way

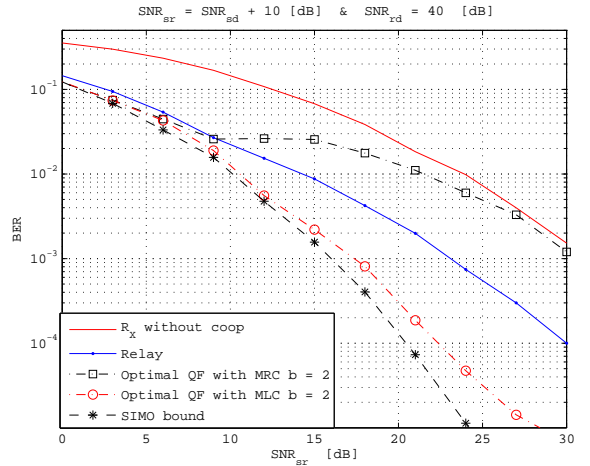


Fig. 4. Case of quasi-static channels: MLD versus MRC

of implementing a channel optimized AF-type protocol in a digital relay transceiver.

Acknowledgments: the authors would like to thank Prof. Pierre Duhamel for many constructive and critical comments.

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