Algorithms for Old Master Painting Canvas Thread Counting from X-rays

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***Abstract*—The task of determining the weave density in the canvas support of Old Master paintings is introduced as a period extraction problem. Because of the way paintings were commonly prepared and preserved, the threads in the horizontal and vertical directions in the canvas support can be counted from x-ray images of the painting. Current procedures are tedious, time-consuming, and (usually) insufficiently documented. This paper describes the design of an algorithm for counting threads from x-rays that uses the Fourier Transform of the Radon Transform of a portion of the image with some crude, but appropriate, decision-making. The algorithm is presented as a sequence of refinements based on a simple mathematical model of the available image data: high resolution x-rays of paintings by Vincent van Gogh from the collection of the Van Gogh Museum. Over 900 spot counts were performed manually by a student team at Cornell using a graphical user interface created for this project. These manual counts provide a dataset for evaluating performance of the algorithm. A major goal is to convince art historians of the viability of automated (and semi-automated) counting procedures.**

I. Introduction

As image data acquisition technology has advanced in the past decade, museums have routinely begun to assemble digital libraries of images of their collections. As described in [1], image processors and art historians can now interact on image analysis tasks that support the art historian's mission of painting analysis as well as activities in image acquisition, storage, and database search. The application of image processing to the investigation of visual art works has recently [2] been declared an important future area for signal processing research. As [1] and [2] acknowledge, the "distance" between the areas of art historical painting analysis and signal processing is a daunting impediment to this cross-disciplinary collaboration.

One approach to this collaboration is to provide (possibly primitive) signal processing tools that help "automate" procedures art historians currently conduct in painting analysis with the hope of expanding the reach of this common practice. Determining the density in threads/cm of the weave in the canvas support of a painting, also known as thread counting, is a candidate procedure.

Art historians use thread count information in support of a claim that the canvases on which different paintings are painted are from the same bolt. When combined with knowledge of the artist's common studio practices, this information can justify the conclusion that the paintings are from the same artist [3], or, as in a study [4] of the collaboration of van Gogh and Gauguin, from two artists. Thread counting has been utilized as a major forensic tool in the attribution efforts of the decades-long Rembrandt Research Project [5].

The counting of threads in the painting's original canvas cannot usually be done from the front of the painting as it is covered by paint. Thread counting also cannot be done from the back (for the vast majority of Old Master paintings) because canvasses have often been strengthened by gluing additional canvas to the back. Fortunately, the preparation of canvas in European painting from the 16th through the 19th centuries commonly involved the application of a layer of x-ray absorbent paint to produce a smooth surface on the rough fabric. With radio-absorbent paint thicker in the valleys between threads, a (positive) x-ray of a painting with a simple linen weave reveals a roughly periodic pattern as shown in the enlarged detail from van Gogh's F651 in Fig. 1.

An analogous task (though distinct due to the character of the images) for which period estimation procedures have been developed is that of chain pattern identification in paper used, e.g., in the prints of Rembrandt [6], [7], as well as in identifying fabric structures in textiles [8].

Current manual thread counting procedures are cumbersome, and are therefore typically done only as a specific question arises rather than as a standard method of documentation for entire collections, as could be the case with automated procedures. The x-ray films are commonly mounted on a light box in a composite of the whole painting. A magnifying eyepiece is used to view the threads alongside a 1 or 2 cm ruler. Since the number of threads traversed in exactly 1 or 2 cm is

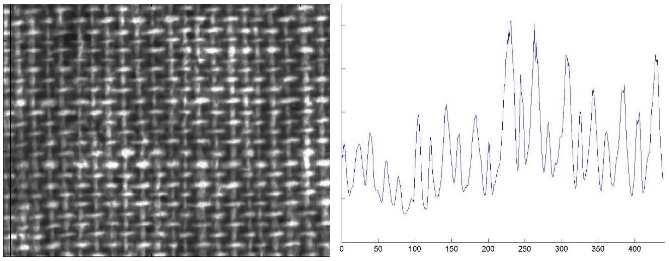

Fig. 1. The left-hand side shows an enlarged detail from a positive print of x-ray negative of van Gogh's "The Walk: Falling Leaves" (F651). Another detail appears in Fig. 3. All thread x-rays are provided courtesy of the Van Gogh Museum. The right-hand side shows the pixel intensities across row 50.

unlikely to be an integer, the remaining fraction is estimated by eye. This makes the preservation of a record of the spots where counts were taken, up to 15 in each – horizontal and vertical – direction, so arduous that it is rarely done. Thus, the current procedure is not repeatable. Such archiving aspects can be readily handled, along with enhanced viewing and manipulation capabilities, once the x-ray images are scanned and thread counting is performed on the computer-generated image. In fact, such record keeping improvements may be as important to the adoption of automated thread counters as the time savings.

Discerning thread density from a digital image is a basic period estimation task to which the Fourier Transform [9] is well suited. The right-hand side of Fig. 1 plots the intensity values from row 50. The Fourier Transform of this signal produces a peak at the location corresponding to 11.84 vertical threads/cm crossing row 50. (The Van Gogh Museum has a value of 11.5 vertical threads/cm recorded in its archives for F651.) However, this crude scheme needs to be modified to deal with the realities of the dataset which consists of x-rays of paintings by Vincent van Gogh scanned at 600 dots per inch and saved as 8-bit greyscale images.

This paper describes an algorithm based on a sequence of refinements to this primitive Fourier-transform-based method. The input is a swatch of an x-ray image (approximately 1 inch square) and the output is an estimate of the thread density. The goal is an accuracy within ± 1 thread/cm 99% of the time. A simple mathematical model of a generic thread pattern is introduced and the goal of the algorithm can be viewed as estimating parameters in that model. Refinements to the model, added to make it represent observed images more closely, are used to guide improvements in the algorithm. Several specific refinements and corresponding improvements are detailed, including use of a Radon transform to determine the rotation compensating for tilt, appropriate data averaging to combat monochromatic "dropout", and "intelligent" rejection of subharmonics due to observed non-uniform thread width patterns. To provide a ground truth against which the algorithm can be tested (which is also needed to convince art historians the algorithm is performing correctly), a team of students at Cornell University counted threads at over 900 spots in over 20 paintings by Vincent van Gogh.

II. A Model of Weave Patterns

A. Idealized Model

Fig. 2 shows an idealized weave pattern consisting of horizontal and vertical ribbons that undulate up and down and intertwine their neighbors. The first assumption is that these undulations are sinusoidal in nature and they occur with (spatial) frequencies f_h and f_v in the horizontal and vertical directions, respectively. Moreover, each set of ribbons alter-

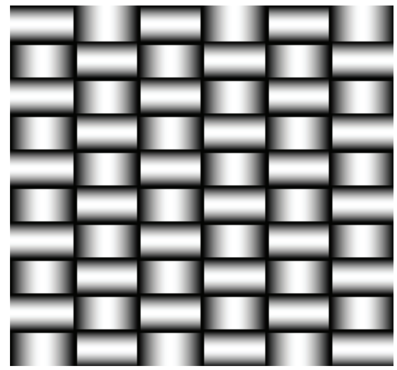

Fig. 2. An idealized weave pattern consists of intertwined horizontal and vertical ribbons. The 6 vertical and 10 horizontal "ribbon" fields are defined in (1) with $f_h = 3$ and $f_v = 5$. The complete image as defined in (2) is the maximum of the vertical and horizontal ribbon fields.

nates in phase. For instance, if the top-most horizontal ribbon is described by $\sin(2\pi f_h x)$, the second horizontal ribbon is phase shifted to $\sin(2\pi f_h x + \frac{\pi}{2})$. Significantly, these phase alternations occur at a rate (frequency) defined by the vertical frequency f_v. Describing this tight coupling between the two frequencies is a key step. Let $\mathrm{square}(z) \triangleq \mathrm{sign}(\sin(z))$ so that $\mathrm{square}(2\pi f_v y)$ represents a square wave with the same frequency as the vertical sinusoid $\sin(2\pi f_v y)$. Then the field of horizontal and vertical ribbons, respectively, are described by

$$\begin{aligned} h(x,y) &= \sin(2\pi f_h x)\,\mathrm{square}(2\pi f_v y) \\ v(x,y) &= -\sin(2\pi f_v y)\,\mathrm{square}(2\pi f_h x). \end{aligned} \tag{1}$$

The two ribbon fields are superimposed so that only the closest (largest) value in each segment remains. This is equivalent to retaining only the maximum value at each point in the image. Hence

$$\begin{aligned} f(x,y) &= \max(h(x,y), v(x,y)) \\ &= \max(\sin(2\pi f_h x)\,\mathrm{square}(2\pi f_v y), \\ &\qquad \sin(2\pi f_v y)\,\mathrm{square}(2\pi f_h x)) \end{aligned} \tag{2}$$

provides a model of this idealized weave pattern as in Fig. 2. Observe that the number of visible ribbons is twice the frequency value.

Of course, real images of weave patterns such as those provided by x-rays of paintings will not be so ideal. There are gaps between successive ribbons, the sinusoidal undulations will not all be the same size, and the ribbons may not be perfectly aligned vertically and horizontally. Each of these imperfections will be modeled in the next subsection.

B. Refined Model

A typical thread x-ray is shown in the left hand side of Fig. 3. This is enhanced using Dagel's thread enhancement algorithm [10] (which conducts a series of morphological operations on the image) to more clearly reveal the underlying ribbon fields, as shown on the right hand side. All three kinds of nonidealities are present: the image is clearly tipped, the ribbons are not all the same size, and there are gaps between adjacent rows and columns.

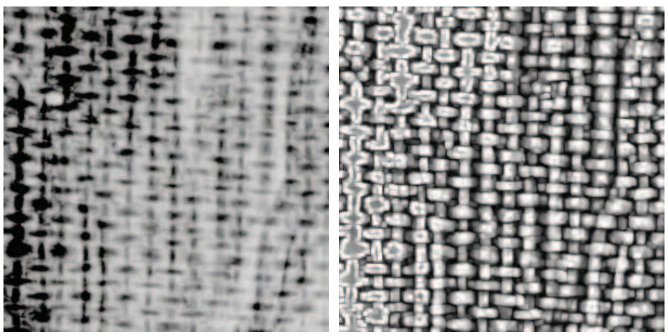

Fig. 3. A segment of a thread x-ray from van Gogh's "The Sheep-Shearer" (F634) and its enhancement using Dagel's thread enhancement algorithm.

Since the problem of thread counting is closely linked to the determination of the frequency parameters f_h and f_v, it is also worthwhile to look at the 2-D Discrete Fourier Transform (DFT). Fig. 4 shows the magnitude of the DFTs of the thread x-ray and the DFT of the enhanced version from Fig. 3. Indeed, the most intense spots in the DFT occur at the true thread frequencies, as we would expect.

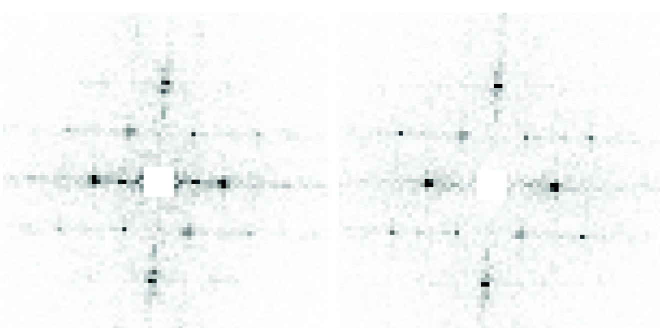

Fig. 4. The magnitude spectra of the thread x-ray and its enhanced version from Fig. 3.

Gaps can be added between adjacent ribbons by creating a dead zone in the square wave whenever it changes values. A constant δ defines the width of the dead zone by

$$\mathrm{square}_\delta(z) = \begin{cases} 0 & |\sin(z)| < \delta \\ \mathrm{sign}(\sin(z)) & \text{otherwise} \end{cases}$$

and the function $\mathrm{square}_\delta(\cdot)$ can be used in (1)-(2) in place of $\mathrm{square}(\cdot)$. For example, the left hand side of Fig. 5 shows the same idealized weave pattern as in Fig. 2 with a small δ added in both the horizontal and vertical dimensions.

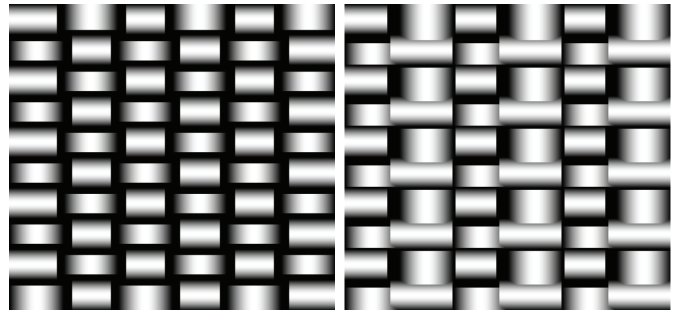

Fig. 5. Dead zones of size 3 and 4 pixels are added to the idealized weave model of Fig. 2. Duty cycles of 40% and 60% are incorporated into the idealized weave model of Fig. 2.

Changing widths of threads can be modeled by changing the duty cycle of the square wave, which is defined to be the percent of the period during which the wave is positive. Changing the duty cycle away from 50% changes the widths of the adjacent ribbons and gives the field more variety of shapes and sizes. The example in the right hand side of Fig. 5 shows the ideal weave with duty cycles of 40% in the horizontal and 60% in the vertical direction. Finally, the image can be rotated a small amount by interpolating adjacent pixel values.

Putting these three kinds of nonidealities together leads to figures such as in Fig. 6, which has many of the features of the enhanced thread x-ray.

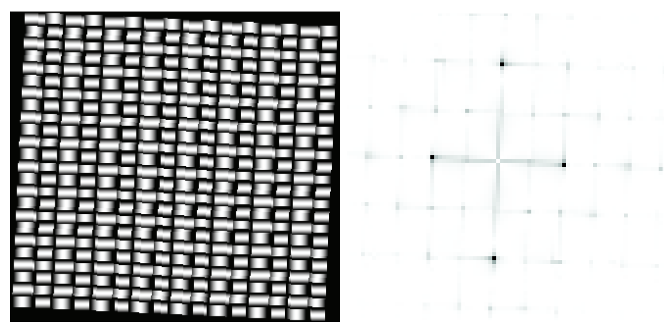

Fig. 6. Using the model, it is possible to replicate many of the features of an enhanced thread x-ray. The left hand side of this figure is generated from the model with 12 vertical and 8 horizontal ribbons. Compare to the enhanced x-ray on the right hand side of Fig. 3. The DFT also duplicates many of the most salient features of the DFTs of the originals.

III. Analysis of Weave Model

One way of determining the amount of rotation of a weave pattern is to project through the image, summing up the values at each angle of projection. Such projections are called the Radon transform [9][11], and this is shown in Fig. 7 for a synthetic weave pattern with a rotation of $5°$.

The vertical slice in Fig. 7 at the rotation angle (in this case, the arrow pointing to the $5°$ slice) is plotted in Fig. 8. This projection can be described mathematically in terms of the idealized weave model of Section II-A. To be concrete,

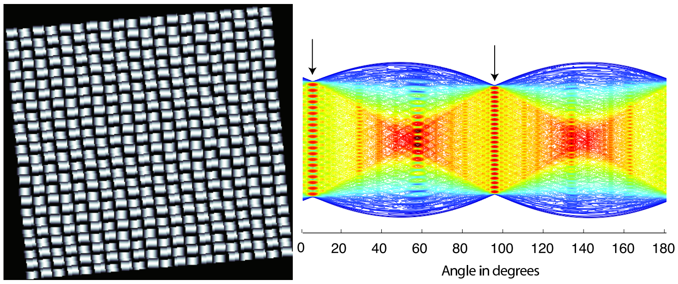

Fig. 7. The weave pattern on the left is rotated by 5 degrees. The radon transform shown on the right projects through the weave pattern at every angle (from $0°$ to $180°$) and sums the pixel values. The display shows large values in red, through yellow to small values in blue. The arrows pointing to the narrowest slices are readily discernible as those with the largest variance, which occurs because the projection at these angles alternately intersects all the ribbons and (nearly none) of the ribbons. These can be seen in this image at $5°$ and at $95°$.

the vertical projection is represented in terms of the model as

$$p(y) = \int f(x,y)dx = \int \max(h(x,y), v(x,y))dx \tag{3}$$

where the horizontal and vertical functions $h(x,y)$ and $v(x,y)$ are defined in (1).

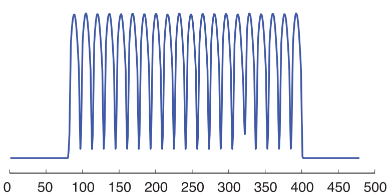

Fig. 8. The projection at $5°$ through the weave pattern in Fig. 7 has a shape that can be calculated from the model.

Observe that the sin term in $h(x,y)$ and the square term in $v(x,y)$ always have the same sign. Similarly, the sin term in $v(x,y)$ and the square term in $h(x,y)$ also have the same sign. Hence $h(x,y)$ is nonnegative whenever $v(x,y)$ is nonpositive and $v(x,y)$ is nonnegative whenever $h(x,y)$ is nonpositive. Accordingly,

$$\begin{aligned}\max(h(x,y), v(x,y)) \quad &= \\ & 1_{\{h(x,y)>0\}}h(x,y) + 1_{\{v(x,y)>0\}}v(x,y)\end{aligned}$$

where

$$1_{z>0} = \begin{cases} 1 & z > 0 \\ 0 & z \leq 0 \end{cases}$$

is the indicator function. Hence the integral (3) can be rewritten as the sum

$$\begin{aligned}p(y) = &\int_{h(x,y)>0} \sin(2\pi f_h x)\ \mathrm{square}(2\pi f_v y)dx \\ &+ \int_{v(x,y)>0} -\sin(2\pi f_v y)\ \mathrm{square}(2\pi f_h x)dx.\end{aligned}$$

Assuming the special case when f_h and f_v are such that an integer number of cycles fit in the image, the first integral is a constant for any value of y

$$\int_{h(x,y)>0} \mathrm{rectify}\{\sin(2\pi f_h x)\}dx = c_v,$$

where $\mathrm{rectify}\{z\} = z1_{z>0}$ is the half-wave rectification function. The second integral can also be calculated under the same assumption as

$$\begin{aligned}\int_{v(x,y)>0} &-\sin(2\pi f_v y)\ \mathrm{square}(2\pi f_h x))dx \\ &= \sin(2\pi f_v y)\int_{v(x,y)>0} -\mathrm{square}(2\pi f_h x))dx \\ &= b_v|\sin(2\pi f_v y)|.\end{aligned}$$

Accordingly,

$$p(y) = c_v + b_v|\sin(2\pi f_v y)| \tag{4}$$

which is directly comparable with the numerical calculation in Fig. 8. An analogous argument shows that the projection in the x direction is

$$p(x) = c_h + b_h|\sin(2\pi f_h x)|.$$

Thus the projections of a derotated weave image directly contain information about the frequency of the threads. In the more general case where an integral number of cycles of f_h and f_v do not fit in the image, $\int_{h(x,y)>0} h(x,y)$ takes on two values (one for the 'even' part of each cycle and one for the 'odd' part of each cycle) and $\int_{v(x,y)>0} v(x,y)$ becomes a weighted combination of two terms of the form $|\sin(2\pi f_v y)|$.

IV. Description of a Semi-Automated Algorithm

This section describes an algorithm for semi-automated thread counts that employs the Radon transform (for derotation to ensure appropriate thread alignment) in combination with the 1-D Fourier Transform (for determinantion of the thread periodicity). Derotation of the image is necessary for two reasons: the x-ray foils may not have been perfectly aligned on the scanner during digitization, and it is also quite common that the horizontal and vertical threads are not perfectly orthogonal (i.e. $90°$ to one another). As discussed above in the caption for Fig. 7, the variance of the Radon transform over a range of projection angles is expected to be largest when the threads are exactly vertically (resp. horizontally) aligned. The angle at which the Radon has the largest variance determines the best angle to align the threads. Without loss of generality, we will assume in the following that vertical threads are being counted.

In any painting x-ray, some regions will be better than others as candidate locations for performing the thread count. It is typical for art historians to avoid counting threads very near the edges, due to scalloping and stretching of the canvas. In addition, certain pigments can be opaque on the resulting x-ray, thus reducing the visibility of the underlying weave. Rather than approach the problem of fully automated counting, we consider a semi-automated scheme where a human operator has selected a line segment along a horizontal thread over which over which the number of vertical threads may be counted. It is assumed that the endpoints of the line start and stop on the center of a whole crossing thread; thus, the total number of threads over the user-selected line is expected to be an integer, and the assumption used to arrive at (4) is valid. The inputs to the algorithm are the digital x-ray image and the coordinates (x_1, y_1) and (x_2, y_2) of the two endpoints over which the thread count is to be taken. The steps of the algorithm are as follows:

Step 1: Extract a circular region of the image which encompasses the user-selected line segment. Assuming the user-selected line segment is approximately m pixels in length, the resulting extracted circular image is contained in an $m \times m$ matrix, where values outside the circle of radius $m/2$ are set to zero.

Step 2: Perform the Radon transform over a range of angles. The maximum rotation angle encountered in the Van Gogh Museum dataset is about $\pm 5°$. Hence the Radon transform is calculated for 101 different angles $\theta \in [-5, -4.9, \ldots, 5]$ degrees. The output of the Radon transform is a $m \times 101$ matrix $\boldsymbol{R}$, representing the 101 different projections at each angle.

Step 3: Choose the rotation angle with the largest variance. The projection with the largest variance is an m-dimensional column from $\boldsymbol{R}$ denoted $\boldsymbol{r}$.

Step 4: Filter the resulting projection. This optional step helps to reduce the effects of high frequency noise. A zero-delay forwards/backwards low-pass filter is used so that the location of the peak is not moved.

Step 5: Take the Fourier Transform of the Radon Transform. Ideally, the filtered projection is something of the form of (4). Consequently, the task is reduced to a one-dimensional period estimation problem (i.e. that of estimating f_v), for which the DFT is well suited.

Step 6: Choose frequency bin with largest magnitude. The frequency bin of the Fourier Transform with largest magnitude is the estimate of f_v. The $n = 9$ lowest frequencies are not considered, however, as there may be a significant (though irrelevant) DC or low-frequency component which dominates the peak of interest.

The number of threads per centimeter is then calculated directly from the estimate of the thread frequency over a line segment of known length.

V. Results and Conclusion

In parallel with the algorithm development, we have developed a graphical user interface (GUI) for testing the algorithm on the dataset provided by the Van Gogh Museum. Using the GUI, a team of Cornell undergraduates manually counted threads over 983 line segments spanning the entire dataset of painting x-rays. Each of these thread counts were independently, blindly verified by another student, and are thus assumed to be 100% accurate. The algorithm of Section IV was tested on all 983 points, and it was found that the estimated thread counts were within ± 0.5 threads per centimeter 84% of the time, and within ± 1 thread per centimeter 95% of the time. When compared with manual thread count measurements provided by the Van Gogh Museum, the algorithm was found to be as accurate [12]. In addition, the semi-automated algorithm provides repeatability and checkability.

Of course, there is still room for improvement in semi-automated thread counting. With further refinement of the model, and further refinement of the corresponding algorithm, we are optimistic that thread counting algorithm performance can be improved. In addition, the use of fully automated algorithms is also of interest, a topic which has recently been explored in [13]. The ability to assemble thread count data that provides a "map" of local counts across the canvas and the cataloging of thread count data for a wide range of van Gogh's paintings have already had a substantial impact on the preparation of the next catalog of late French period paintings by Vincent van Gogh in the collection of the Van Gogh Museum.

Acknowledgements

The authors would like to thank the Van Gogh Museum for making the digital x-ray images available. In addition, we thank the team of Cornell undergraduates, as well as Jon San Ng for performing the painstaking manual counts which were instrumental in verifying algorithm performance.

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