Average Age of Information in Update Systems with Active Sources and Packet Delivery Errors

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***Abstract*—This paper studies the "age of information" (AoI) in a multi-source status update system where N active sources each send updates of their time-varying process to a monitor through a server with packet delivery errors. We analyze the average AoI for stationary randomized and round-robin scheduling policies. For both of these scheduling policies, we further analyze the effect of packet retransmission policies, i.e., retransmission without resampling, retransmission with resampling, or no retransmission, when errors occur. Expressions for the average AoI are derived for each case. It is shown that the round-robin schedule policy in conjunction with retransmission with resampling when errors occur achieves the lowest average AoI among the considered cases. For stationary randomized schedules with equiprobable source selection, it is further shown that the average AoI gap to round-robin schedules with the same packet management policy scales as $\mathcal{O}(N)$. Finally, for stationary randomized policies, the optimal source selection probabilities that minimize a weighted sum average AoI metric are derived.**

***Index Terms*—Age of information, multi-source, active sources, scheduling, packet transmission errors.**

I. Introduction

Freshness of information is of critical importance in networked monitoring and control systems like intelligent vehicular systems. The *Age of Information* metric was first proposed in [1] to capture the timeliness of received information. In the simplest setting a source sends updates to a destination through a channel that is typically modeled as a server with random service time, e.g., [2]–[6]. The multi-source and/or multi-destination setting with error-free packet delivery was considered in [7]–[20].

Several recent papers have considered the effect of packet delivery errors on AoI [21]–[32]. The single-source single-destination setting was first studied in [21]. The average AoI was derived for scheduled access with feedback and slotted ALOHA-like random access over multiaccess channels in [23]. A single-source multi-destination setting was considered in [25] where a single base station source sends status updates to a number of destinations through packets with a fixed transmission time over unreliable channels. It was shown that a greedy policy, which schedules a transmission to the destination with the highest current age is average age optimal in the absence of error. In [26], [31], the opposite setting was studied where a network of nodes transmits status updates to a base station while simultaneously satisfying throughput constraints. In this setting, stationary randomized policies are derived to minimize the weighted sum AoI assuming a unit-delay channel from each source to the destination. A lower bound was derived on the average peak AoI in multi-hop networks with time-slotted transmissions and packet loss [32].

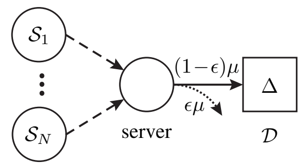

Fig. 1. The multi-source status update system with unreliable transmissions.

This paper considers a multi-source single-destination status update system where the sources send information packets to a destination through a server with unreliable transmissions. The setting considered in this paper is shown in Fig. 1 and is similar to the multi-source single-destination setting considered in [23], [26]. A key difference, however, is that the service times in our model are assumed to be random according to the exponential distribution. Another key difference is that we consider both stationary randomized policies as well as round-robin policies under three different packet retransmission policies when errors occur. Table I summarizes the key differences between the results in this paper compared to [21], [23], [25], [26]. The main contributions of this paper are twofold: (i) we derive closed-form expressions for each source's long-term average AoI from the perspective of the destination, and (ii) we derive optimal source selection probabilities that minimize the weighted sum average AoI for stationary randomized policies.

II. System Model

We consider a status update system with N sources $\mathcal{S}_i$ for $i \in \mathbb{I} = \{1,...,N\}$ and one destination node $\mathcal{D}$ as shown in Fig. 1. The sources intend to share information about their local time-varying state with the destination. We assume that the time required to sample a status update is negligible and that each source can generate packets containing status updates "at will" as in [24], [27]. The packets are sent to the destination through a server with service time $S \overset{\text{i.i.d.}}{\sim} \text{Exp}(\mu)$. We further assume that upon service completion, a packet is lost with probability ϵ. The destination sends instantaneous and error-free feedback to the server after every service completion indicating whether the transmission was successful or not. The following sections

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TABLE I
COMPARING THE AVERAGE AOI ANALYSIS IN [21], [23], [25], [26] WITH THIS WORK.

	single/multi-source/destination	status update arrival rate at the source(s)	service rate	stationary randomized policy			round-robin policy		
				no retransmission	retransmission w/o resampling	retransmission w/ resampling	no retransmission	retransmission w/o resampling	retransmission w/ resampling
[21]	single-source single-destination	Poisson	Exponential	—	—	—	—	—	—
[23]	multi-source single-destination	at the beginning of each time slot	constant / time-slotted	—	—	✓	—	—	✓
[25]	single-source multi-destination	at the beginning of each frame	constant / time-slotted	✓	—	—	—	✓	—
[26]	multi-source single-destination	at will / instantaneously	constant / time-slotted	✓	—	—	—	—	—
This work	multi-source single-destination	at will / instantaneously	Exponential	✓	✓	✓	✓	✓	✓

discuss the two scheduling policies considered here: (i) the stationary randomized policy and (ii) the round-robin policy.

A. Stationary Randomized Schedule Policy

We assume a fixed probability mass function $\mathcal{P} = \{p_1, ..., p_N\}$ with $p_i > 0$ corresponding to the probability that source $\mathcal{S}_i$ is selected to transmit. We denote the indices of the transmitted packets over time by $j \in \mathbb{J} = \{1, 2, ...\}$ and $m_j \in \mathbb{I}$ as the source of the j^{th} packet. If the j^{th} packet is successfully delivered, the next source m_{j+1} is simply drawn randomly from $\mathcal{P}$. If the j^{th} packet is lost, then we consider three packet management approaches:

A1. **No Retransmission**: The server ignores errors and simply draws source m_{j+1} from $\mathcal{P}$ as if no error occurred. We refer to this case as "RND_NR".

A2. **Retransmission Without Resampling**: The server sets $m_{j+1} = m_j$ and retransmits the *original* packet from $\mathcal{S}_{m_j}$. We refer to this case as "RND_ARQ".

A3. **Retransmission With Resampling**: The server sets $m_{j+1} = m_j$ and transmits a *fresh* packet from $\mathcal{S}_{m_j}$. We refer to this case as "RND_ASQ".

B. Round-Robin Schedule Policy

For the round-robin scheduling policy, transmitting nodes are selected deterministically in order. If the j^{th} packet is successfully delivered, the next source is $m_{j+1} = \{m_j \bmod N\} + 1$. If the j^{th} packet is lost, then we consider same three packet management approaches as in the stationary randomized case. These packet management approaches are denoted as 'RR_NR" (no retransmission, errors are ignored), "RR_ARQ" (the original packet from $\mathcal{S}_{m_j}$ is retransmitted until successfully delivered), and "RR_ASQ" (a fresh packet from $\mathcal{S}_{m_j}$ is transmitted until successfully delivered), respectively.

C. Average Age Metric

The age $\Delta_i(t)$ of the status of $\mathcal{S}_i$ at the destination is a linearly increasing random process when no updates arrive at the destination and has downward jumps when an update is received. The average age of the status updates of $\mathcal{S}_i$ from the perspective of the destination is defined as [2]

$$\Delta_i \triangleq \lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} \Delta_i(t)\ dt. \tag{1}$$

III. AVERAGE AGE ANALYSIS FOR STATIONARY RANDOMIZED AND ROUND-ROBIN POLICIES

In this section we analyze the average AoI for the two scheduling policies in Section II-A and Section II-B.

A. Stationary Randomized Schedule Policy

Theorem 1 presents the average AoI for the stationary randomized schedule policies.

Theorem 1. *The average AoI Δ_i of the status updates of source $i \in \mathbb{I}$ for the multi-source system with active sources and service completion with error probability ϵ under the stationary randomized policies is equal to*

$$\Delta_{i,\text{RND\_NR}} = \frac{1 + (1-\epsilon)p_i}{\mu(1-\epsilon)p_i}, \tag{2a}$$

$$\Delta_{i,\text{RND\_ARQ}} = \frac{1 + p_i}{\mu(1-\epsilon)p_i}, \tag{2b}$$

$$\Delta_{i,\text{RND\_ASQ}} = \Delta_{i,\text{RND\_NR}}. \tag{2c}$$

Proof sketch. We use tools from Stochastic Hybrid Systems (SHS) [16] to derive the average age. Due to space limitations, most of the algebraic derivations are omitted here. A Markov chain representation of the discrete state $q(t) \in \mathcal{Q}$ of the system regarding $\Delta_i(t)$ for case RND_NR is shown in Fig. 2. Table II represents the exponential rate and the transition map for each link ℓ with continuous state $[x_0, x_1]$, where x_0 represents the age of the $\mathcal{S}_i$'s state at $\mathcal{D}$ and x_1 stores the age to be used after an age reset when $\mathcal{S}_i$ successfully delivers a packet (link 0). For notational convenience, we denote

$$\mathbf{D}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ \mathbf{D}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},\ \text{and } \mathbf{D}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{3}$$

Since there is only one state in Fig. 2, the stationary distribution of the Markov chain is trivial and we can write the single balance equation (Theorem 4, [16]) as

$$\mu\bar{\mathbf{v}}_0 = \mathbf{b} + \lambda^{(0)}\bar{\mathbf{v}}_{q_0}\mathbf{A}_0 + \lambda^{(1)}\bar{\mathbf{v}}_{q_1}\mathbf{A}_1, \tag{4}$$

Fig. 2. The Markov chain of the status update system in Fig. 1 regarding $\Delta_i(t)$ for case RND_NR. Link $\ell = 0$ corresponds to a successfully delivered packet for $\mathcal{S}_i$. Link $\ell = 1$ corresponds to an unsuccessfully delivered packet for $\mathcal{S}_i$ as well as a successful or unsuccessful packet from any other source.

TABLE II
TRANSITION RATES AND MAPS FOR CASE RND_NR.

ℓ	$q_\ell \to q'_\ell$	$\lambda^{(\ell)}$	$\mathbf{x}\mathbf{A}_\ell$	$\mathbf{A}_\ell$	$\mathbf{v}_{q_\ell}\mathbf{A}_\ell$
0	$0 \to 0$	$\mu(1-\epsilon)p_i$	$[x_1, 0]$)	$\mathbf{D}_1$	$[v_{01}, 0]$
1	$0 \to 0$	$\mu(1-(1-\epsilon)p_i)$	$[x_0, 0]$)	$\mathbf{D}_0$	$[v_{00}, 0]$

where $\mathbf{b} = [1, 1]$ and $\bar{\mathbf{v}} = [\bar{v}_{00}, \bar{v}_{01}]$. Substituting the quantities from Table II and solving for v_{00} yields

$$\bar{v}_{00} = \frac{1 + (1-\epsilon)p_i}{\mu(1-\epsilon)p_i}, \tag{5}$$

which shows (2a).

We use a similar analysis for the RND_ARQ and RND_ASQ cases. Both of these cases can be represented by the Markov chain in Fig. 3. The system enters state 0 after any successful transmission. The system enters state 1 after an unsuccessful transmission from $\mathcal{S}_i$ and the system enters state 2 after an unsuccessful transmission from $\mathcal{S}_j$ for $j \neq i$. Links 0 and 4 correspond to successful transmissions by $\mathcal{S}_i$. Links 2 and 3 correspond to unsuccessful transmissions by $\mathcal{S}_i$. The remaining links correspond to successful and unsuccessful transmissions from $\mathcal{S}_j$ for all $j \neq i$.

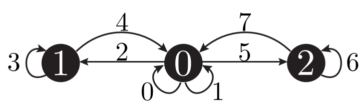

Fig. 3. The Markov chain for cases RND_ARQ and RND_ASQ.

Table III shows the transition rates for case RND_ARQ. The table for RND_ASQ is identical except for links 2 and 3 where, for RND_ASQ, we have $\mathbf{A}_2 = \mathbf{A}_3 = \mathbf{D}_0$ and $\mathbf{x}\mathbf{A}_2 = \mathbf{x}\mathbf{A}_3 = [x_0, 0]$. For RND_ASQ we also have $\mathbf{v}_{q_2}\mathbf{A}_2 = [v_{00}, 0]$ and $\mathbf{v}_{q_3}\mathbf{A}_3 = [v_{10}, 0]$. These differences are due to the fact that, since RND_ASQ always transmits a fresh sample, links 2 and 3 reset the stored age in x_1. A similar analysis as above can be applied to solve for the steady state distribution of the Markov chain as

$$\bar{\boldsymbol{\pi}} = [\bar{\pi}_0 \ \bar{\pi}_1 \ \bar{\pi}_2] = \begin{bmatrix} 1-\epsilon & \epsilon p_i & \epsilon(1-p_i) \end{bmatrix}. \tag{6}$$

and then solving the balance equations for $\mathbf{v}_0$, $\mathbf{v}_1$, and $\mathbf{v}_2$ and then compute $\Delta_i = v_{00} + v_{10} + v_{20}$ to arrive at (2b) and (2c).

In general, for any fixed system parameters, we have $\Delta_{i,\text{RND\_ARQ}} \geq \Delta_{i,\text{RND\_NR}} = \Delta_{i,\text{RND\_ASQ}}$. The fact that the achieved average age is identical between cases RND_NR and RND_ASQ can be understood intuitively by noting that the destination always receives a *fresh* sample with both RND_NR and RND_ASQ. Moreover, the rates of successful packets from $\mathcal{S}_i$ are the same in both cases, i.e., for RND_ASQ, the sum of links 0 and 4 weighted by the steady state probabilities of the Markov chain can be computed as $\bar{\pi}_0\lambda^{(0)} + \bar{\pi}_1\lambda^{(4)} =$

TABLE III
TRANSITION RATES AND MAPS FOR CASE RND_ARQ.

ℓ	$q_\ell \to q'_\ell$	$\lambda^{(\ell)}$	$\mathbf{x}\mathbf{A}_\ell$	$\mathbf{A}_\ell$	$\mathbf{v}_{q_\ell}\mathbf{A}_\ell$
0	$0 \to 0$	$\mu(1-\epsilon)p_i$	$[x_1, 0]$	$\mathbf{D}_1$	$[v_{01}, 0]$
1	$0 \to 0$	$\mu(1-\epsilon)(1-p_i)$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{00}, 0]$
2	$0 \to 1$	$\mu\epsilon p_i$	$[x_0, x_1]$	$\mathbf{D}_2$	$[v_{00}, v_{01}]$
3	$1 \to 1$	$\mu\epsilon$	$[x_0, x_1]$	$\mathbf{D}_2$	$[v_{10}, v_{11}]$
4	$1 \to 0$	$\mu(1-\epsilon)$	$[x_1, 0]$	$\mathbf{D}_1$	$[v_{11}, 0]$
5	$0 \to 2$	$\mu\epsilon(1-p_i)$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{00}, 0]$
6	$2 \to 2$	$\mu\epsilon$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{20}, 0]$
7	$2 \to 0$	$\mu(1-\epsilon)$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{20}, 0]$

$\mu(1-\epsilon)^2 p_i + \mu\epsilon(1-\epsilon)p_i = \mu(1-\epsilon)p_i$, which is the same as link 0 in RND_RR.

B. Round-robin policy

Theorem 2 presents the average AoI for the round-robin schedule policies.

Theorem 2. *The average AoI Δ_i of the status updates of source $i \in \mathbb{I}$ for the multi-source system with active sources and service completion with error probability ϵ under the round-robin policies is equal to*

$$\Delta_{i,\text{RR\_NR}} = \frac{N + 3 + (N-3)\epsilon}{2\mu(1-\epsilon)}, \tag{7a}$$

$$\Delta_{i,\text{RR\_ARQ}} = \frac{N+3}{2\mu(1-\epsilon)}, \tag{7b}$$

$$\Delta_{i,\text{RR\_ASQ}} = \frac{N + 3 - 2\epsilon}{2\mu(1-\epsilon)}. \tag{7c}$$

Proof sketch. A Markov chain and transition rates for case RR_NR regarding $\Delta_1(t)$ are shown in Fig. 4 and Table IV, respectively. State m corresponds to $\mathcal{S}_m$ selected to transmit. Links 0 and 1 correspond to successful and unsuccessful transmissions from $\mathcal{S}_1$, respectively. The remaining links are for transmissions from $\mathcal{S}_j$ for all $j \neq 1$ and do not distinguish between successful or unsuccessful transmissions.

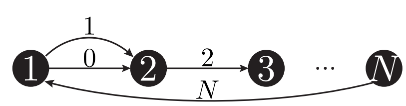

Fig. 4. The Markov chain for case RR_NR.

TABLE IV
TRANSITION RATES AND MAPS FOR CASE RR_NR.

ℓ	$q_\ell \to q'_\ell$	$\lambda^{(\ell)}$	$\mathbf{x}\mathbf{A}_\ell$	$\mathbf{A}_\ell$	$\mathbf{v}_{q_\ell}\mathbf{A}_\ell$
0	$1 \to 2$	$\mu(1-\epsilon)$	$[x_1, 0]$	$\mathbf{D}_1$	$[v_{11}, 0]$
1	$1 \to 2$	$\mu\epsilon$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{10}, 0]$
i	$i \to i+1$	μ	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{i0}, 0]$
N	$N \to 1$	μ	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{N0}, 0]$

Both RR_ARQ and RR_ASQ can be represented by the Markov chain in Fig. 5. Here, the even numbered links correspond to unsuccessful transmissions and the odd numbered links correspond to successful transmissions. The transition rates for case RR_ARQ are shown in Table V. Similar to the previous discussion, the only difference between RR_ARQ and RR_ASQ is that the stored age x_1 is reset in link 0 for

RR_ASQ. Hence, the transition rate table for case RR_ASQ is the same as Table V except for link 0 where, for RND_ASQ, we have $\mathbf{A}_0 = \mathbf{D}_0$, $\mathbf{x}\mathbf{A}_0 = [x_0, 0]$, and $\mathbf{v}_{q_0}\mathbf{A}_0 = [v_{10}, 0]$. A similar analysis as above can be applied to solve for $\mathbf{v}_1, \ldots, \mathbf{v}_N$ and then compute $\Delta_i = \sum_{i=1}^{N} v_{i0}$ to get (7a)-(7c).

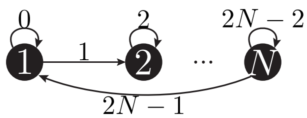

Fig. 5. The Markov chain for cases RR_ARQ and RR_ASQ.

TABLE V
TRANSITION RATES AND MAPS FOR CASE RR_ARQ, $i \in \{2, \ldots, N-1\}$.

ℓ	$q_\ell \to q'_\ell$	$\lambda^{(\ell)}$	$\mathbf{x}\mathbf{A}_\ell$	$\mathbf{A}_\ell$	$\mathbf{v}_{q_\ell}\mathbf{A}_\ell$
0	$0 \to 0$	$\mu\epsilon$	$[x_0, x_1]$	$\mathbf{D}_2$	$[v_{10}, v_{11}]$
1	$0 \to 1$	$\mu(1-\epsilon)$	$[x_1, 0]$	$\mathbf{D}_1$	$[v_{11}, 0]$
$2i-2$	$i \to i$	$\mu\epsilon$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{i0}, 0]$
$2i-1$	$i \to i+1$	$\mu(1-\epsilon)$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{i0}, 0]$
$2N-2$	$N \to N$	$\mu\epsilon$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{N0}, 0]$
$2N-1$	$N \to 1$	$\mu(1-\epsilon)$	$[x_0, 0]$	$\mathbf{D}_0$	$[v_{N0}, 0]$

C. Discussion

To compare the RND and the RR policies, we can assume $p_1 = \ldots = p_N = \frac{1}{N}$. From (2a)-(2c) and (7a)-(7c) we have

$$\Delta_{i,\text{RND\_NR}} - \Delta_{i,\text{RR\_NR}} = \frac{N-1}{2\mu} \geq 0, \tag{8a}$$

$$\Delta_{i,\text{RND\_ARQ}} - \Delta_{i,\text{RR\_ARQ}} = \frac{N-1}{2\mu(1-\epsilon)} \geq 0, \tag{8b}$$

$$\Delta_{i,\text{RND\_ASQ}} - \Delta_{i,\text{RR\_ASQ}} = \frac{N-1}{2\mu(1-\epsilon)} \geq 0. \tag{8c}$$

The average age gap between RND and RR policies can be intuitively understood by considering the case when $\epsilon = 0$. In this case, the round-robin policy ensures each source is regularly sampled whereas a randomized stationary policy, even when sampled in the same overall proportion as the round-robin schedule, samples each source irregularly. This irregular sampling causes an increase in the average age with respect to the round-robin schedule.

IV. Optimal Randomized Stationary Policy for Minimizing Weighted Sum Average AoI

In this section we find the optimal source selection probabilities $p_1^*, \ldots, p_N^*$ that minimize the general weighted sum average AoI among all stationary randomized policies. Considering Theorem 1, this problem can be formulated as

$$\min_{p_i} \text{WSAoI}, \qquad \text{s.t.} \sum_{i=1}^{N} p_i = 1, \tag{9}$$

where

$$\text{WSAoI}_{\text{RND\_NR}} = \text{WSAoI}_{\text{RND\_ASQ}} \triangleq \sum_{i=1}^{N} \frac{\alpha_i[1+(1-\epsilon)p_i]}{\mu N(1-\epsilon)p_i}, \tag{10a}$$

$$\text{WSAoI}_{\text{RND\_ARQ}} \triangleq \sum_{i=1}^{N} \frac{\alpha_i[1+p_i]}{\mu N(1-\epsilon)p_i}, \tag{10b}$$

and $\alpha_i \geq 0$ denotes the fixed wight for source i. Without loss of generality we assume $\sum_{i=1}^{N} \sqrt{\alpha_i} = 1$. Theorem 3 represents the optimal solution for the problem in (9).

Theorem 3. *For the RND policies, the optimal p_i is*

$$p^*_{i,\text{RND\_NR}} = p^*_{i,\text{RND\_ARQ}} = p^*_{i,\text{RND\_ASQ}} = \sqrt{\alpha_i}. \tag{11}$$

Proof: Considering (10a)-(10b), the Hessian matrix of WSAoI can be written as

$$\mathbf{H}(\text{WSAoI}) = \text{diag}\left(\frac{2\alpha_1}{\mu N(1-\epsilon)p_1^3}, \ldots, \frac{2\alpha_N}{\mu N(1-\epsilon)p_N^3}\right). \tag{12}$$

Since $\alpha_i > 0$ we have $|\mathbf{H}(\text{WSAoI})| > 0$, which means that WSAoI is a convex function of $p_1, \ldots, p_N$ and there exist a set of optimal p_i values that minimize WSAoI. From (10a) we define the following Lagrangian multiplier function

$$\mathcal{L}(p_i, \lambda) = \frac{1}{N}\sum_{i=1}^{N} \frac{\alpha_i[1+p_i(1-\epsilon)]}{\mu p_i(1-\epsilon)} + \lambda(\sum_{i=1}^{N} p_i - 1). \tag{13}$$

Taking the partial derivative of (13), we get

$$\frac{\partial \mathcal{L}(p_i, \lambda)}{\partial p_i} = -\frac{\alpha_i}{N\mu(1-\epsilon)p_i^2} + \lambda, \tag{14a}$$

$$\frac{\partial \mathcal{L}(p_i, \lambda)}{\partial \lambda} = \sum_{i=1}^{N} p_i - 1. \tag{14b}$$

Setting (14a) and (14b) to zero, we get

$$p^*_{i,\text{RND\_NR}} = p^*_{i,\text{RND\_ASQ}} = \sqrt{\alpha_i}, \quad \lambda = \frac{1}{N\mu(1-\epsilon)}. \tag{15}$$

Repeating steps (13)-(14b) for (10b) gives $p^*_{i,\text{RND\_ARQ}}$. □

V. Numerical Results

This section presents numerical examples to illustrate and verify the achieved average AoI. Figure 6 represents the average age pairs (Δ_1, Δ_2) for $N=2$ sources, error probability $\epsilon = \{0.15, 0.6\}$ and normalized service rate $\mu = 1$. The results show that $\Delta_1 + \Delta_2$ is minimized under the round-robin policy with retransmissions of fresh samples.

Figure 7 represents WSAoI versus p_1 for $\alpha_1 = 0.49$, $\alpha_2 = 0.09$, $N=2$, $\epsilon = 0.6$ and $\mu = 1$. Since the information from source $\mathcal{S}_1$ has a higher weight, intuitively over the long term more packets from $\mathcal{S}_1$ should be delivered to the destination to minimize WSAoI. The simulation results show that the minimum WSAoI is reached when $p^*_{1,\text{RND\_NR}} = p^*_{1,\text{RND\_ARQ}} = p^*_{1,\text{RND\_ASQ}} = 0.7$, which agrees with Theorem 3. For the two extreme cases where $p_1 \to 0$ ($p_2 \to 1$) and $p_1 \to 1$ ($p_2 \to 0$) we have $\Delta_1 \to \infty$ (Δ_2 becomes finite) and Δ_1 becomes finite ($\Delta_2 \to \infty$), respectively, giving WSAoI$\to\infty$.

VI. Conclusion

This paper analyzed the average AoI for a multi-source status update system with packet delivery errors. For two scheduling policies, we derived simple closed-form expressions for the average AoI under three different packet management approaches whenever errors occur. The round-robin policy with retransmission of fresh samples was shown to have the lowest average AoI among the considered cases. The gap

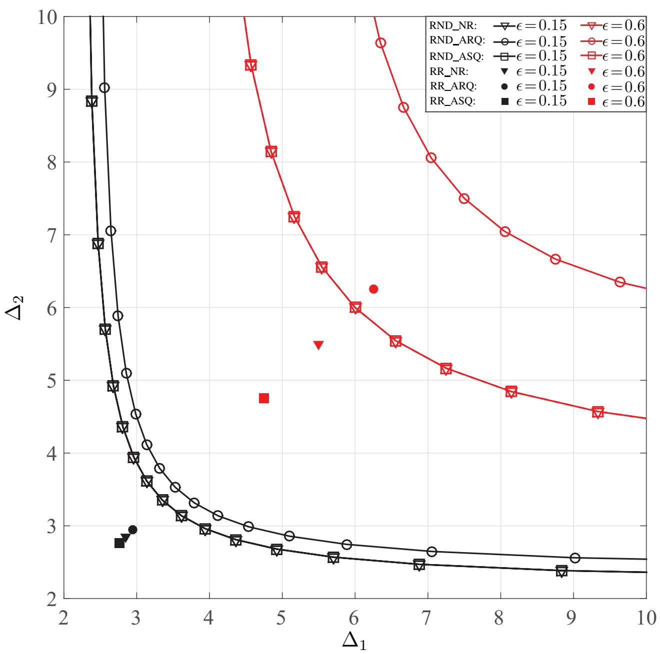

Fig. 6. Average age pairs (Δ_1, Δ_2) for $N=2$, $\epsilon=\{0.15, 0.6\}$ and $\mu=1$.

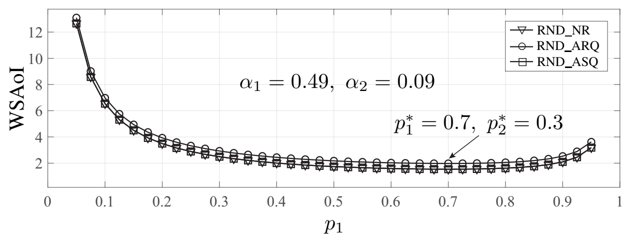

Fig. 7. Weighted sum average age versus p_1 for $N = 2$, $\epsilon = 0.6$ and $\mu = 1$.

between the randomized stationary and round-robin policies was also shown to scale with $\mathcal{O}(N)$. For a general problem where the sources have different priorities, the source selection probabilities were optimized to minimize the weighted sum average AoI for stationary randomized policies. Future directions of this work include deriving fundamental bounds on the AoI of stationary randomized policy and developing an optimal scheduling policy that minimizes the age.

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