On the Outage Performance of Relay Systems in Frequency Selective Fading Channels

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***Abstract*—We consider a three-node wireless relay system with frequency selective quasi-static fading channels between the nodes. We assume practical constraints require the relay to operate in half-duplex mode and employ a low-complexity non-orthogonal amplify-forward protocol. Assuming full channel state information and optimal energy allocation between source and relay, we study upper bounds of the outage probability for such channels. Compared to relay systems in flat fading channels, our results reveal that the additional diversity offered by frequency selective fading channels significantly improves outage performance. The results also suggest, however, that relays may offer a limited improvement in diversity when multipath fading in the source-destination link is sufficiently rich.**

I. Introduction

In wireless communications, common channel impairments that limit capacity include multipath fading, shadowing, and path loss. Diversity is a powerful technique to combat fading and increase reliability. There are three basic diversity techniques: (1) time diversity, (2) space diversity, and (3) frequency diversity. Spatial diversity is particularly attractive since it provides diversity gain without using additional time or bandwidth resources [1]. Cooperative relays and multiple antennas are two ways to exploit spatial diversity. We are interested in relay cooperation for its applicability to small devices in ad-hoc networks which may not be able to employ multiple antennas due to size or complexity limitations. Extensive research has been done for flat fading relay channels (e.g. [1]–[4]), and it has been shown that the maximum diversity order [5] of non-orthogonal amplify forward (NAF) protocol with a single relay is two [4].

For high data-rate wireless applications, intersymbol interference (ISI) is often encountered since the symbol duration is small compared to the delay spread of the multipath channel [5]. In *static* frequency selective channels, ISI is typically considered to be undesirable and various signal design techniques and equalization techniques have been developed to reduce ISI. In frequency selective *fading* channels, however, the ISI results in an increased diversity order with maximum value equal to the number of fading coefficients in the channel [6]. Several works have considered frequency selective fading in relay channels, including [7]–[9]. Both [7] and [8] considered orthogonal forwarding relay model in which delay was introduced at the relays and both showed that relaying with certain delay could increase the diversity order of the system. In [7], the delay at the relays was motivated by the inaccuracy at symbol-level timing synchronization. In [8], the authors showed that the diversity order was increased since the controlled time delays at the relays increased the overall frequency selectivity seen from the destination side. In [9], the authors considered both full-duplex and half-duplex relay channel models with a decode-forward protocol at the relay. Constrained ergodic information rate bounds were calculated and simulation-based results showed that the relay channel with frequency selective fading has a higher information rate than the relay channel with flat fading.

In this paper, we are interested in assessing the outage probability of relay systems used in channels with frequency selective fading. With emphasis on simplicity and spectral efficiency, we address the use of the non-orthogonal amplify and forward (NAF) relay protocol through quasi-static frequency selective fading channels where the relays operate under half-duplex constraint. Our goal is to study the minimum outage probability of the relay system under a long-term average total transmit energy constraint with energy control over the source and the relay. We seek to provide insight into the following questions: (1) What is the effect of frequency selectivity on the outage probability in systems employing NAF relays? (2) Does a relay provide the same performance improvement in the frequency selective fading relay channel compared with the flat fading relay channel?

II. System Model

As illustrated in Fig. 1, the wireless communication system consists of three nodes: source node (S), relay node (R), and destination node (D). The links among the terminals are modeled as complex finite impulse response (FIR) filters with each channel coefficient having i.i.d. Rayleigh fading statistics [5]. The source to destination channel coefficients are represented as $\boldsymbol{h}_{sd}$ with length N_{sd}, the source to relay channel coefficients as $\boldsymbol{h}_{sr}$ with length N_{sr}, and the relay to destination channel coefficients as $\boldsymbol{h}_{rd}$ with length N_{rd}. We assume complex zero-mean additive white Gaussian noise $\boldsymbol{w}_r$ and $\boldsymbol{w}_d$ with corresponding variances σ_r^2 and σ_d^2 at the relay and the destination, respectively. The channel coefficients are assumed to be constant over a block during which one codeword is transmitted, and are independent from one block to the other. We assume the message sent from the transmitter at the source node is encoded to a block $\boldsymbol{x}$ of L source symbols. We further assume that the transmission energy at

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the source is subjected to the block average energy constraint $\mathcal{E}_s$ and the relay to $\mathcal{E}_r$ where $\mathcal{E}_s + \mathcal{E}_r \leq \mathcal{E}$, which is the total average energy available for the source and the relay.

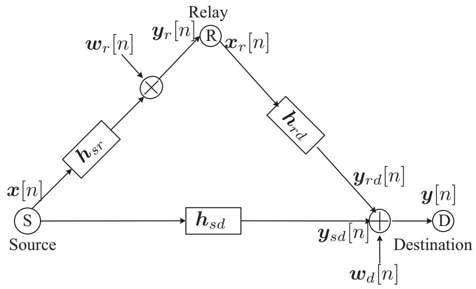

Fig. 1. Relay channel model with frequency selective fading

We assume the relay operates in a half-duplex mode, and thus does not transmit and receive at the same time. In the NAF protocol, the on-off behavior of the relay effectively introduces a frame structure into the composite signal received at the destination. The frame transmission time is divided into two equal portions, each of length N. Thus, a block of L symbols consists of many frames of length $2N$. A frame sent by the source can be expressed as $2N$ symbols $x[1], x[2], \cdots, x[2N]$. During the first time slot, the relay works as a receiver while the source transmits $x[1], x[2], \cdots, x[N]$ to both the relay and the destination. During the second time slot, the relay works as a transmitter; the source continues transmitting symbols $x[N+1], x[N+2], \cdots, x[2N]$ to the destination, while the relay simultaneously transmits a scaled version of what it received in the first time slot to the destination. Thus, during the second time slot, the destination receives a superposition of signals from the source and the relay.

In an ISI channel, the delayed reflections of the symbols sent by the source in the first time slot may affect the received signal at the destination during the second time slot. Also, the delayed reflections of the previous frame may affect the received signal in the current frame duration. The choice of frame length $2N$ is irrelevant in situations with flat fading where the channel has no memory, and it has not been considered in the existing literature. Frame length selection may play a role in the performance of systems in ISI channels, however.

The signal $\boldsymbol{y}_r \in \mathbb{C}^{L+N_{sr}-1}$ received by the relay is expressed as $\boldsymbol{y}_r = \boldsymbol{H}_{sr}\boldsymbol{x} + \boldsymbol{w}_r$ where $\boldsymbol{x} \in \mathbb{C}^L$ is the source symbols, $\boldsymbol{w}_r$ is complex Gaussian noise vector with covariance $\sigma_r^2 \boldsymbol{I}_{L+N_{sr}-1}$ and $\boldsymbol{H}_{sr} \in \mathbb{C}^{(L+N_{sr}-1)\times L}$ is the Tœplitz channel convolution matrix defined by $[\boldsymbol{H}_{sr}]_{i,j} = h_{sr}[j-i]$. Before the relay forwards the signal to the destination, the relay scales $\boldsymbol{y}_r$ by β so that the symbols to be sent by the relay satisfy the block average energy constraint $\mathcal{E}_r$. The signal sent by the relay is written as $\boldsymbol{x}_r = \beta \boldsymbol{\Gamma}_r \boldsymbol{y}_r$ where

$$\boldsymbol{\Gamma}_r = \boldsymbol{I}_{(L+N_{sr}-1)/(2N)} \otimes \begin{pmatrix} \boldsymbol{0}_{N\times N} & \boldsymbol{0}_{N\times N} \\ \boldsymbol{I}_N & \boldsymbol{0}_{N\times N} \end{pmatrix}.$$

The role of $\boldsymbol{\Gamma}_r$ is to apply the half-duplex constraint since it zeros samples in the second time slot and shift samples of $\boldsymbol{y}_r$ from the first time slot to be the input of relay to destination channel in the second time slot. The relay amplification factor β is chosen to satisfy the relay energy constraint $E[\boldsymbol{x}_r^H \boldsymbol{x}_r] \leq L\mathcal{E}_r$.

Finally, the received signal at the destination is then expressed as the input-output relation

$$\boldsymbol{y} \quad = \quad \underbrace{\boldsymbol{H}_{sd}\boldsymbol{x}}_{\text{from source}} + \underbrace{\boldsymbol{H}_{rd}\beta\boldsymbol{\Gamma}_r \overbrace{(\boldsymbol{H}_{sr}\boldsymbol{x} + \boldsymbol{w}_r)}^{\boldsymbol{y}_r}}_{\text{from relay}} + \boldsymbol{w}_d \qquad (1)$$

where $\boldsymbol{H}_{rd} \in \mathbb{C}^{(L+N_{sr}+N_{rd}-2)\times(L+N_{sr}-1)}$ and $\boldsymbol{H}_{sd} \in \mathbb{C}^{(L+N_{sd}-1)\times L}$ are Tœplitz channel convolution matrices containing the random channel coefficients $\boldsymbol{h}_{rd}$ and $\boldsymbol{h}_{sd}$, and $\boldsymbol{w}_d$ is complex zero-mean Gaussian noise vector with covariance matrix $\sigma_d^2 \boldsymbol{I}_{L+N_{sd}-1}$. By defining

$$\boldsymbol{H}_{eff} \quad \triangleq \quad \boldsymbol{H}_{sd} + \beta \boldsymbol{H}_{rd}\boldsymbol{\Gamma}_r \boldsymbol{H}_{sr} \qquad (2)$$

we can rewrite the input-output relation from (1) as

$$\boldsymbol{y} \quad = \quad \boldsymbol{H}_{eff}\boldsymbol{x} + \boldsymbol{w} \qquad (3)$$

where $\boldsymbol{w}$ is colored noise with covariance

$$\boldsymbol{R}_{ww} = \sigma_d^2 \boldsymbol{I} + \beta^2 \sigma_r^2 \boldsymbol{H}_{rd}\boldsymbol{\Gamma}_r \boldsymbol{\Gamma}_r^T \boldsymbol{H}_{rd}^H.$$

We note $\boldsymbol{H}_{sd}$, $\boldsymbol{H}_{sr}$ and $\boldsymbol{H}_{rd}$ are all lower triangular Tœplitz matrices and $\boldsymbol{\Gamma}_r$ is a block diagonal matrix with the same lower triangular diagonal block. We can show the effective channel matrix $\boldsymbol{H}_{eff}$ and the noise covariance matrix $\boldsymbol{R}_{ww}$ are both block Tœplitz matrices. In addition, we note that the L-block average energy constraint can be shown to be equivalent to a stricter $2N$-frame average energy constraint. While we omit the proof due to space constraints, the steps are very similar to the proof shown in [10].

III. Outage Probability

In this section, we calculate an achievable rate, and investigate a upper bound on the outage probability of the system model described in the previous section. The outage probability $P_{out}(R)$ is defined as the probability that the mutual information $I(\boldsymbol{x};\boldsymbol{y})$ between the transmitted signal and received signal is below a certain transmission rate R. The outage probability provides a lower bound on the block error-rate performance and quantifies the potential gain of the relay scheme [11]. In order to compute the achievable rate, it is assumed that all nodes have complete channel state information (CSI), i.e. each node knows the instantaneous values of the channel coefficients as well as their statistics. It is furthermore assumed that all nodes are perfectly synchronized. The outage probability of systems with relays employing the NAF protocol has been well studied for the case of flat-fading channels [1], but here we study ISI channels which are frequency selective. We also assume the total average energy $\mathcal{E}$ is allocated optimally among the terminals based on the channel states in order to maximize the achievable rate for each channel state. We define

$$P_{out}(R, \mathcal{E}) = \inf_{\mathcal{E}_r, \mathcal{E}_s : \mathcal{E}_r + \mathcal{E}_s \leq \mathcal{E}} \mathcal{P}(C < R)$$

with

$$C = \lim_{L\to+\infty} \sup_{\substack{q_L(\boldsymbol{x}),\beta:\operatorname{tr}(\boldsymbol{K_x})\le L\mathcal{E}_s,\\ \beta^2\operatorname{tr}(\boldsymbol{\Gamma}_r\boldsymbol{K}_{\boldsymbol{y}_r}\boldsymbol{\Gamma}_r^H)\le L\mathcal{E}_r}} \frac{1}{L} I(\boldsymbol{x};\boldsymbol{y}) \quad (4)$$

where $\boldsymbol{K_x} = E[\boldsymbol{x}\boldsymbol{x}^H]$, $\boldsymbol{K}_{\boldsymbol{y}_r} = \boldsymbol{H}_{sr}\boldsymbol{K_x}\boldsymbol{H}_{sr}^H + \sigma_r^2\boldsymbol{I}_{L+N_{sr}-1}$, and L is the block length. The supremum in (4) of the mutual information $I(\cdot)$ is taken jointly over all probability densities $q_L(\boldsymbol{x})$ for the sequence $\boldsymbol{x}$ satisfying the source block average energy constraint $\operatorname{tr}(\boldsymbol{K_x}) \le L\mathcal{E}_s$ and the relay energy constraint $\beta^2\operatorname{tr}(\boldsymbol{\Gamma}_r\boldsymbol{K}_{\boldsymbol{y}_r}\boldsymbol{\Gamma}_r^H) \le L\mathcal{E}_r$. Since we consider a non-memoryless system, it is also assumed that the initial state of the system is the zero state. The choice of the initial state will not affect the achievable rate of the system if the length of the system memory is finite.

At first glance, solving (4) for the simple system model of the form (3) would seem to be a fairly straightforward extension of results from classic point-to-point ISI channels where water-filling results in the optimal input distribution [10]. Computation of (4), however, is complicated by the fact that the relay has its own energy constraint which is embedded in the effective channel. As the source distribution changes, so, too, does the received signal energy observed at the relay due to the fact that the relay only receives half of the time. Since the effective channel in (2) depends on the relay amplification factor β, which in turn depends on the received signal energy observed at the relay, *the effective channel matrix itself is a function of the source distribution*. Consequently, we are unable to solve (4) directly with classical water-filling.

A. Maximum rate for a fixed β

Due to the difficulty in solving (4), we consider a suboptimal approach which leads to an achievable rate. To temporarily remove the dependence of the effective channel coefficients on the relay amplification factor β at the relay, we begin by first choosing a fixed value of β denoted as β^*, thereby ignoring the relay energy constraint. With β^* and fixed energy allocation, the supremum in (4) can be written

$$C_L = \sup_{q_L(\boldsymbol{x}):\operatorname{tr}(\boldsymbol{K_x})\le L\mathcal{E}_s^*} \frac{1}{L} I(\boldsymbol{x};\boldsymbol{y}) \quad (5)$$

where we temporarily ignore the relay energy constraint $\beta^{*2}\operatorname{tr}(\boldsymbol{\Gamma}_r\boldsymbol{K}_{\boldsymbol{y}_r}\boldsymbol{\Gamma}_r^H) \le L\mathcal{E}_r^*$. Since the noise is independent of the input, the information rate is maximized when the distribution of $\boldsymbol{y}$ is Gaussian. As the relation between $\boldsymbol{x}$ and $\boldsymbol{y}$ is linear, the maximum information rate is achieved when the input distribution is also Gaussian. The conventional method to find the supremum in (5) is to decompose the effective channel into parallel channels and employ water-filling. Before decomposing the channel, however, we whiten the noise in $\boldsymbol{y}$. As $\boldsymbol{R}_{ww}$ is always symmetric positive definite, there exists a decomposition (e.g. Cholesky) $\boldsymbol{R}_{ww} = \boldsymbol{G}\boldsymbol{G}^H$ where $\boldsymbol{G}$ is invertible and is called the whitening filter. Applying this whitening filter $\boldsymbol{G}$ to $\boldsymbol{y}$, the overall channel matrix becomes $\boldsymbol{G}^{-1}\boldsymbol{H}_{eff}$ and the additive noise becomes white. Decomposing this channel with SVD as $\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}^H = \boldsymbol{G}^{-1}\boldsymbol{H}_{eff}$, the maximum rate with only a source energy constraint is given by the classical water-filling solution [10]

$$C_L = \frac{1}{L}\sum_{i=0}^{L-1}\log_2[\max(\Theta|\lambda_i|^2, 1)] \quad (6)$$

where the λ_i are the singular values of the whitened channel (i.e. the diagonal elements of $\boldsymbol{\Lambda}$) and Θ is the solution of

$$\sum_{i=0,\lambda_i\neq 0}^{L-1} \underbrace{\max\left(\Theta - |\lambda_i|^{-2}, 0\right)}_{\triangleq \sigma_i} = L\mathcal{E}_s^*. \quad (7)$$

B. Iterative approach to finding an achievable rate

The water-filling solution assumes β^* is fixed and thus ignores the relay energy constraint. Consequently, it may result in a source distribution which violates relay energy constraint $\beta^{*2}\operatorname{tr}(\boldsymbol{\Gamma}_r\boldsymbol{K}_{\boldsymbol{y}_r}\boldsymbol{\Gamma}_r^H) \le L\mathcal{E}_r^*$. Let $\boldsymbol{\Sigma}$ be a diagonal matrix with σ_i as the ith diagonal element as defined in (7). The optimal input covariance with water-filling is $\boldsymbol{K}_{\boldsymbol{x}}^* = \boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{V}^H$, and the transmission energy consumed by the relay with the chosen β^* is $\beta^{*2}\operatorname{tr}(\boldsymbol{\Gamma}_r\boldsymbol{K}_{\boldsymbol{y}_r}^*\boldsymbol{\Gamma}_r^H)$. If $\beta^{*2}\operatorname{tr}(\boldsymbol{\Gamma}_r\boldsymbol{K}_{\boldsymbol{y}_r}^*\boldsymbol{\Gamma}_r^H) \le L\mathcal{E}_r^*$, the energy constraint at the relay is satisfied and the source covariance $\boldsymbol{K}_{\boldsymbol{x}}^*$ found through water-filling could be used as a possible input covariance. Even if the relay energy constraint is satisfied, however, there is no guarantee that this source distribution and relay amplification factor are the global optimizers of (5).

For a given source covariance $\boldsymbol{K_x}$, it is trivial to adjust the relay amplification factor β so that it satisfies the relay energy constraint with equality by setting $\beta' = \sqrt{\frac{L\mathcal{E}_r^*}{\operatorname{tr}(\boldsymbol{\Gamma}_r\boldsymbol{K}_{\boldsymbol{y}_r}^*\boldsymbol{\Gamma}_r^H)}}$. The source covariance $\boldsymbol{K_x}$, however, was optimized under the old choice β. This suggests an iterative algorithm where we guess a value of β, calculate $\boldsymbol{K_x}$ via water-filling, adjust the relay amplification factor to β', calculate a new $\boldsymbol{K_x}$ via water-filling, re-adjust the relay amplification factor, and continue in this manner in hopes of converging to a pair $(\boldsymbol{K_x}, \beta)$ which satisfies the relay energy constraint with equality.

Thus, a candidate iterative method to find a lower bound of the capacity in (4) for a fixed energy allocation is comprised of the following steps:

1) **Initialization**. Set $\beta_0 = \sqrt{\frac{L\mathcal{E}_r^*}{\frac{L+N_{sr}-1}{2}\sigma_r^2}}$, set the maximum number of iterations M_I, and set a stopping tolerance $\epsilon > 0$.
2) **Perform water-filling to calculate source covariance**. At ith iteration, and with relay amplification factor β_i, compute the noise covariance as $\boldsymbol{R}_{ww,i} = \boldsymbol{G}_i\boldsymbol{G}_i^H$, compute the SVD of the whitened channel matrix $\boldsymbol{G}_i^{-1}\boldsymbol{H}_{eff,i} = \boldsymbol{U}_i\boldsymbol{\Lambda}_i\boldsymbol{V}_i$, and compute the water-filling solution $\boldsymbol{\Sigma}_i$ as given in (7). Then compute the covariance matrices $\boldsymbol{K}_{\boldsymbol{x},i} = \boldsymbol{V}_i\boldsymbol{\Sigma}_i\boldsymbol{V}_i^H$ and

$$\boldsymbol{K}_{\boldsymbol{y}_r,i} = \boldsymbol{H}_{sr}\boldsymbol{K}_{\boldsymbol{x},i}\boldsymbol{H}_{sr}^H + \sigma_r^2\boldsymbol{I}_{L+N_{sr}-1}.$$

3) **Adjust $\boldsymbol{\beta}$**. Adjust the relay amplification factor so that

it meets the relay energy constraint via

$$\beta_{i+1} = \sqrt{\frac{L\mathcal{E}_r^*}{\mathrm{tr}(\boldsymbol{\Gamma}_r \boldsymbol{K}_{\boldsymbol{y}_r,i} \boldsymbol{\Gamma}_r^T)}}.$$

Note that the pair $(\boldsymbol{K}_{\boldsymbol{x},i}, \beta_{i+1})$ are "valid" in the sense that they satisfy all energy constraints. The pair $(\boldsymbol{K}_{\boldsymbol{x},i}, \beta_i)$, on the other hand, are not guaranteed to be valid, as they may violate the relay energy constraint.

4) **Check stopping condition**.
   - If $i > M_I$, pick the maximum in the valid set of achievable rates, algorithm ends.
   - Else if $|\beta_i - \beta_{i+1}| \leq \epsilon$, algorithm has converged. Compute the achievable rate, algorithm ends;
   - Else if $\beta_i < \beta_{i+1}$, compute the achievable rate, put it into the valid set. Note that in this case, the pair $(\boldsymbol{K}_{\boldsymbol{x},i}, \beta_i)$ results in a situation where the relay does not use all its available energy, yet the pair provides an achievable rate. Increment i, go to step 2).
   - Else if $\beta_i > \beta_{i+1}$, the pair $(\boldsymbol{K}_{\boldsymbol{x},i}, \beta_i)$ is not valid. Increment i, go to step 2).

If the algorithm ends at the stopping criterion of $|\beta_i - \beta_{i+1}| \leq \epsilon$, the algorithm has converged. In this case, the pair $(\boldsymbol{K}_{\boldsymbol{x},i}, \beta_i)$ results in a situation where the relay uses all its available energy, and the source distribution satisfies the water-filling criterion. The motivation for this stopping criterion is that we expect the maximum achievable rate to occur when the relay uses all of its available energy. We have observed through simulation that the convergence speed of this algorithm is relatively fast. For example, with randomly generated channel states $\boldsymbol{h}_{sd} = [0.1324 - 0.2477j]$,

$$\boldsymbol{h}_{sr} = [-0.529 - 0.890j, 0.884 + 0.851j, -0.315 - 0.764j]^\top$$
$$\boldsymbol{h}_{rd} = [-0.670 - 0.131j, 0.715 - 0.778j, -0.755 + 0.312j]^\top$$

when $\epsilon = 0.001$, $\mathrm{SNR_d} = \mathrm{SNR_r} = 0\mathrm{dB}$, block length $L = 100$, and $\mathcal{E}_s = \mathcal{E}_s = 0.5$, the value of β goes through $79.4328, 0.5645, 0.7796, 0.7792$ and the achievable rate goes through $\mathrm{NA}, \mathrm{NA}, 9.5733, 9.7686$ where NA denotes that the current amplification factor and source covariance are a not valid pair because the relay energy constraint is not satisfied. For this example, the algorithm terminates after four iterations. Even when the algorithm converges to $(\boldsymbol{K}_{\boldsymbol{x},i}, \beta_i)$, it is not guaranteed to find the supremum in (4); in fact, there may be other valid pairs $(\boldsymbol{K}_{\boldsymbol{x},j}, \beta_j)$ for $j < i$ found during the algorithm iteration which yield a higher achievable rate, as mentioned in the third bullet of step 4.

The additional stopping criterion of a maximum number of iterations M_I is needed as $|\beta_i - \beta_{i+1}|$ might always be greater than ϵ due to the possibility of limit cycles which we have observed (rarely) for certain channel matrices. We save all calculated achievable rates corresponding to valid amplification factor and source covariance pairs, and if the algorithm does not stop in M_I iterations, we choose the largest valid rate at algorithm termination, in which case the relay does not use all the available energy.

C. Energy allocation

The iterative algorithm proposed in the previous section finds a lower bound of the capacity for fixed energy allocation. As stated, the algorithm either stops with an achievable rate when the relay does not use all the available energy or converges with an achievable rate which is possibly less than the achievable rate when the relay does not use all the available energy. It is possible to exploit the unused available energy to increase the achievable rate with energy allocation. To find an approximately optimal energy allocation, we employ a grid search with $\mathcal{E}_s = \alpha\mathcal{E}$ and $\mathcal{E}_r = (1-\alpha)\mathcal{E}$ where $0 \leq \alpha \leq 1$. In [12], it was shown that the optimal energy allocation for the flat-fading NAF relay channel results in either direct transmission or orthogonal relaying. This same problem was investigated by [13] through simulation and our simulation result for flat-fading channels is consistent with the results shown in figure 5 of [13]. As flat-fading channels are a special case of frequency-selective fading channels, this consistency suggests that under optimal energy allocation, the maximum achievable rate should be obtained when the relay uses all the available energy. It should be emphasized that the iterative algorithm converges when the relay uses all the available energy.

IV. Numerical Results

To illustrate the performance of the relay model, we consider a finite length block size of $L = 100$ with relay period $2N = 2$. While our theoretical results assume the block length is infinite, we found that beyond a sufficiently large block size (e.g. $L \geq 100$), the numerical results do not noticeably differ. Since our objective here is to assess the performance gain afforded by a frequency selective channel, the channel taps in the source-relay, source-destination, and relay-destination channels are all assumed to have the same power decay and i.i.d. Rayleigh fading. The signal-to-noise ratio (SNR) at the relay SNR_r is the same as that at the destination SNR_d.

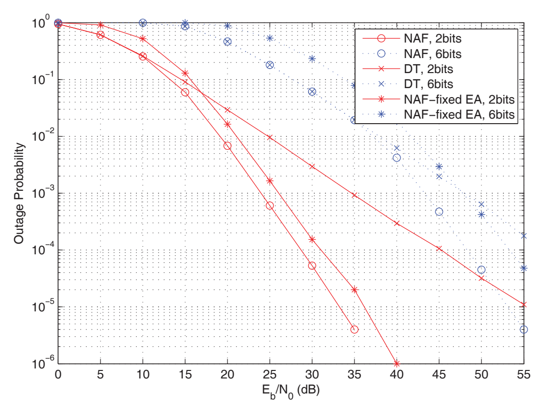

Fig. 2. Outage probability with all links having flat fading

First, for comparison, we present the well-studied case [1] where the source-relay link, source-destination link and relay-destination link all experience flat fading. We note that our frequency-selective model is a generalization of the flat-fading case, and that flat-fading is obtained by choosing the channel lengths to all have a single tap. In Fig. 2 we observe the well-known result that the NAF protocol with one relay yield diversity order that is twice that of simple direct transmission [1]. The diversity order d is the absolute value of the slope of the outage curve. Note that for our numerical results, d is calculated numerically as the slope of the least-squares fit through the linear portion of the outage curve.

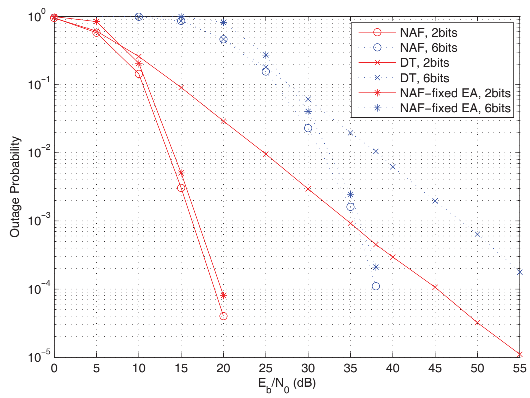

Fig. 3. Outage probability with relay links having frequency selective fading

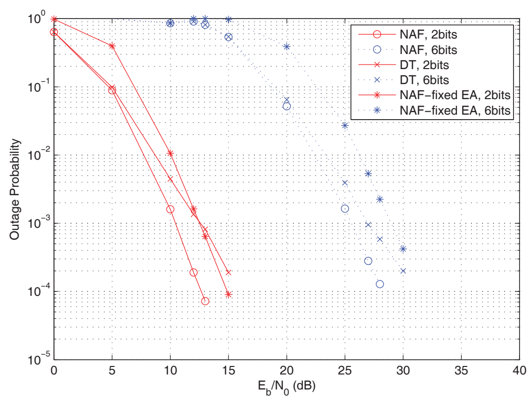

Fig. 4. Outage probability with all links having frequency selective fading

Next, we consider the case where the source-destination link experiences flat fading but the source-relay link and relay-destination link have frequency selective fading with order 3. As shown in Fig. 3, the diversity order of the NAF scheme is about doubled when the source-relay link and relay-destination link are frequency selective fading when compared with the diversity order of the NAF scheme with purely flat-fading shown in Fig. 2. Thus, the presence of additional fading taps in the source-relay-destination path significantly increases the diversity order of the system.

We then consider a case where the source-destination link, source-relay link, and relay-destination link all consist of 3rd order frequency selective fading channels. As can be seen from Fig. 4, the additional channel fading taps result in yet more diversity, as is to be expected. However, we observe that direct transmission (i.e. the case with no relay) also experiences a significant performance gain with a diversity order about 3 as there are three fading taps in the source-destination channel, as can be seen by comparing Fig. 4 with Fig. 2.

It is also interesting to observe that the increase of diversity order of NAF protocol over direct transmission in Fig. 4 is less than that in Fig. 3, which demonstrates that if the source-destination link is already frequency selective (i.e. if it already has sufficient diversity), the performance improvement provided by the relay is less than when the direct link is flat fading.

V. Conclusion

In conclusion, we have developed the system model and proposed an iterative algorithm to calculate an achievable rate for this system model. We have investigated the outage probability for NAF relays in slowly-fading frequency-selective channels under a variety of conditions, and our results suggest the additional diversity offered by frequency-selective fading can signicantly improve outage performance. We also observed, however, that systems operating in frequency-selective channels that already have significant diversity in the source-destination link the may reap limited benefit in using relay transmission.

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